1a $\Delta x = 6 - 2 = 4$ (km) en $\Delta y = 9{,}6 - 5 = 4{,}6$ (€).

1b $\frac{4{,}6}{4} = 1{,}15$ (€ / km) $= \frac{\Delta y}{\Delta x}$.

1c $5 - 2 \cdot 1{,}15 = 5 - 2{,}30 = 2{,}70$ (€).

1d Uit 1b en 1c volgt: $y = 1{,}15x + 2{,}70$.

2a $5q - 12 = 90 - 12q$
$7q = 102$
$q = 6$.

2b $6x - 3 = 2(3 - x) + 13$
$6x - 3 = 6 - 2x + 13$
$8x = 22 \Rightarrow x = 2{,}75$.

2c $5(2p - 7) = 25 - (5 - p)$
$10p - 35 = 25 - 5 + p$
$9p = 55 \Rightarrow p = \frac{55}{9}$.

2d $\frac{1}{2}y + 18 = \frac{2}{3}y + 5$
$-\frac{1}{6}y = -13 \Rightarrow y = 78$.

2e $11a - 3(2a - 6) = 193 - 2a$
$11a - 6a + 18 = 193 - 2a$
$7a = 175 \Rightarrow a = 25$.

2f $6(3 - p) = 5(2p - 1) + 23$
$18 - 6p = 10p - 5 + 23$
$-16p = 0 \Rightarrow p = 0$.

3a $P = aq + b$ met $a = \frac{\Delta P}{\Delta q} = \frac{125 - 57}{17 - 13} = \frac{68}{4} = 17$.
$\left.\begin{array}{l} P = 17q + b \\ q = 13 \Rightarrow P = 57 \end{array}\right\} \Rightarrow 57 = 17 \cdot 13 + b$
$57 - 17 \cdot 13 = b = -164$.
Dus $P = 17q - 164$.

```
57-17*13
                -164
```

3b $A = at + b$ met $a = \frac{\Delta A}{\Delta t} = \frac{300 - 360}{61 - 55} = \frac{-60}{6} = -10$.
$\left.\begin{array}{l} A = -10t + b \\ t = 61 \Rightarrow A = 300 \end{array}\right\} \Rightarrow 300 = -10 \cdot 61 + b$
$300 + 10 \cdot 61 = b = 910$.
Dus $A = -10t + 910$.

```
300+10*61
                 910
```

4ab $K = 1{,}4q + 864$ en $R = 2{,}6q$. (volgt direct uit de gegevens)

4c $K = R$
$1{,}4q + 864 = 2{,}6q$
$-1{,}2q = -864$
$q = 720$ (vazen).

```
864/1.2
                 720
```

4d $W = R - K = 900$
$2{,}6q - (1{,}4q + 864) = 900$
$1{,}2q - 864 = 900$
$1{,}2q = 1764$
$q = 1\,470$ (vazen).

```
900+864
                1764
Ans/1.2
                1470
```

5a $n = aT + b$ met $a = \frac{\Delta n}{\Delta T} = \frac{32 - 24}{24 - 19} = \frac{8}{5} = 1{,}6$.
$\left.\begin{array}{l} n = 1{,}6T + b \\ T = 24 \Rightarrow n = 32 \end{array}\right\} \Rightarrow 32 = 1{,}6 \cdot 24 + b$
$32 - 1{,}6 \cdot 24 = b = -6{,}4$.
Dus $n = 1{,}6T - 6{,}4$.

```
32-1.6*24
                -6.4
```

5b $n = 1{,}6T - 6{,}4$
$n + 6{,}4 = 1{,}6T$
$T = \frac{n + 6{,}4}{1{,}6} = \frac{1}{1{,}6}n + \frac{6{,}4}{1{,}6} = 0{,}625n + 4$.

```
1/1.6
                .625
6.4/1.6
                   4
```

5c $n = \frac{88}{4} = 22 \Rightarrow T = 0{,}625 \cdot 22 + 4 = 17{,}75 \approx 18$ (°C).

```
0.625*22+4
               17.75
```

6a $a = 150$ (per vrachtauto) $\Rightarrow K = 200$ (€ / TEU). Dus de totale kosten zijn $10 \cdot 200 = 2\,000$ euro.

6b 300 TEU per trein voor 69000 euro $\Rightarrow K = \frac{69000}{300} = 230$ (€ / TEU) $\Rightarrow a = 365$ (km).

6c Vrachtauto: lijn door (0, 50) en (50, 100) $\Rightarrow \frac{\Delta K}{\Delta a} = \frac{50}{50} = 1$ en $K = a + 50$.
Trein: lijn door (50, 150) en (250, 200) $\Rightarrow \frac{\Delta K}{\Delta a} = \frac{50}{200} = \frac{1}{4} = 0{,}25$ en $K = 0{,}25a + b$ door (250, 200) $\Rightarrow b = 137{,}5$.
Schip: lijn door (0, 150) en (250, 200) $\Rightarrow \frac{\Delta K}{\Delta a} = \frac{50}{250} = \frac{1}{5} = 0{,}2$ en $K = 0{,}2a + 150$.

```
200-0.25*250
               137.5
```

6d $a + 50 = 0{,}25a + 137{,}5$
$0{,}75a = 87{,}5$
$a = \frac{87{,}5}{0{,}75} \approx 117$.

```
87.5/0.75
         116.6666667
```

$0{,}25a + 137{,}5 = 0{,}2a + 150$
$0{,}05a = 12{,}5$
$a = \frac{12{,}5}{0{,}05} = 250$.

```
12.5/0.05
                 250
```

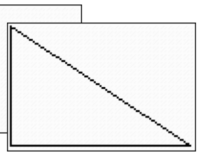
Aflezen in grafiek: voor afstanden tussen 117 en 250 km is per trein het voordeligst

7a $30x + 24y = 15000$
(x keer € 30 en y keer € 24 levert € 15000 op)
$24y = -30x + 15000$
$y = -\frac{30x}{24} + \frac{15000}{24}$
$y = -1{,}25x + 625$. (zie een plot hiernaast)

```
-30/24
               -1.25
15000/24
                 625
```

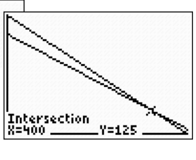
```
Plot1 Plot2 Plot3
\Y1=-1.25X+625
\Y2=
```

```
WINDOW
 Xmin=0
 Xmax=500
 Xscl=0
 Ymin=0
 Ymax=625
 Yscl=0
 Xres=1
```

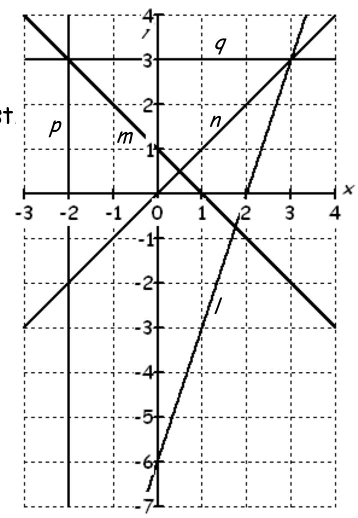


7b $x + y = 525$ (er worden 525 kaarten verkocht)
$y = -x + 525$. (zie de plot hiernaast)

```
Plot1 Plot2 Plot3
\Y1=-1.25X+625
\Y2=-X+525
\Y3=
```



7c $-1{,}25x + 625 = -x + 525$ (intersect) $\Rightarrow x = 400$ en $y = 125$.
Er waren dus 125 pashouders aanwezig.



8ab l: $3x - y = 6$
$x = 0 \Rightarrow y = 6$
$y = 0 \Rightarrow x = 2$
$rc_l = 3$.

m: $x + y = 1$
$x = 0 \Rightarrow y = 1$
$y = 0 \Rightarrow x = 1$
$rc_m = -1$.

n: $x - y = 0$
$x = 0 \Rightarrow y = 0$
$y = 1 \Rightarrow x = 1$
$rc_n = 1$.

p: $y = 3$ (een hor. lijn)
$x = 0 \Rightarrow y = 3$
$x = 1 \Rightarrow y = 3$
$rc_p = 0$.

q: $x = -2$ (een vert. lijn)
$y = 0 \Rightarrow x = -2$
$y = 1 \Rightarrow x = -2$
rc_q bestaat niet.

9a $3x-4y=12$
$-4y=-3x+12$
$y=\frac{3}{4}x-3.$

9b $2x+6=3y-12$
$-3y=-2x-18$
$y=\frac{2}{3}x+6.$

9c $2x+3y=y-20$
$2y=-2x-20$
$y=-x-10.$

9d $2{,}5x-3=-6y+30$
$6y=-2{,}5x+33$
$y=-\frac{5}{12}x+5\frac{1}{2}.$

10a Stel $x=$ het aantal stoelen en $y=$ het aantal tafels $\Rightarrow 350x+850y=22\,000.$

10b Stel $x=$ de prijs van 100 gram cashewnoten en $y=$ de prijs van 100 gram studentenhaver $\Rightarrow 4x+7y=8{,}40.$

10c Stel $x=$ het aantal ha groenten en $y=$ het aantal ha graan $\Rightarrow 10\,000x+5\,000y=250\,000.$

11a $5a-17=3b-9$
$-3b=-5a+8$ (delen door -3)
$b=\frac{5}{3}a-\frac{8}{3}.$

11b $4p+80=-\frac{1}{3}q+125$
$\frac{1}{3}q=45-4q$ (keer 3)
$q=135-12q.$

11c $5(3t-7)=3(7-A)-8$
$15t-35=21-3A-8$
$3A=48-15t$ (delen door 3)
$A=16-5t.$

12a $L=207-0{,}85\cdot170-1{,}02\cdot18=44{,}14\approx44.$

```
207-0.85*170-1.0
2*18
            44.14
```

12b $99=207-0{,}85\cdot120-1{,}02W\Rightarrow1{,}02W=6\Rightarrow W\approx5{,}9.$

```
207-0.85*120-99
                6
Ans/1.02
      5.882352941
```

12c Bij een moeilijk boek zullen S en W groot zijn. Hoe groter S en W, hoe lager (de leesbaarheidsfactor) L is.

13a $y=2x+3$ invullen in $B=5x+3y+8\Rightarrow B=5x+3(2x+3)+8=5x+6x+9+8=11x+17.$

13b $r=5p+8$ invullen in $q=-2p+3r+6\Rightarrow q=-2p+3(5p+8)+6=-2p+15p+24+6=13p+30.$

13c $t=-3p+3\frac{1}{2}\Rightarrow3p=3\frac{1}{2}-t$ invullen in $A=2t+3p+9\Rightarrow A=2t+3\frac{1}{2}-t+9=t+12\frac{1}{2}.$

13d $y=2x+6$ invullen in $A=5xy+20\Rightarrow A=5x(2x+6)+20=10x^2+30x+20.$

14a $t=-\frac{1}{2}p+6\Rightarrow\frac{1}{2}p=6-t\Rightarrow p=12-2t$ in $A=2t+5p+9\Rightarrow A=2t+5(12-2t)+9=-8t+69.$

14b $y=2x+6\Rightarrow2x=y-6\Rightarrow x=\frac{1}{2}y-3$ in $A=6xy+20\Rightarrow A=6y(\frac{1}{2}y-3)+20=3y^2-18y+20.$

14c $p=-3q+6$ en $2q=-r+8\Rightarrow r=8-2q$ in $K=5p+3q+5r+100\Rightarrow K=5(-3q+6)+3q+5(8-2q)+100=-22q+170.$

15a $l=50\Rightarrow BMR=66+13{,}7g+5h-6{,}8\cdot50=13{,}7g+5h-274.$

```
66-6.8*50
             -274
```

15b $l=28$, $g=68$ en $BMR=1700\Rightarrow1700=66+13{,}7\cdot68+5h-6{,}8\cdot28\Rightarrow892{,}8=5h\Rightarrow h\approx179$ (cm).

```
1700-66-13.7*68+
6.8*28
            892.8
Ans/5
           178.56
```

15c $l=40$ en $g=h-100\Rightarrow BMR=66+13{,}7\cdot(h-100)+5h-6{,}8\cdot40=18{,}7h-1576.$

```
66+13.7*-100-6.8
*40
            -1576
```

16a $\frac{1}{x^3}=x^{-3}.$

16b $\sqrt[4]{x^3}=x^{\frac{3}{4}}.$

16c $\sqrt{x}=x^{\frac{1}{2}}.$

16d $\frac{1}{x}=x^{-1}.$

17a $\frac{x^{15}\cdot x^5}{x^3}=\frac{x^{20}}{x^3}=x^{17}.$

17c $x^{2,6}\cdot\frac{1}{x^{3,2}}\cdot\frac{1}{x}=\frac{x^{2,6}}{x^{4,2}}=x^{-1,6}.$

17e $\left(x^{a-1}\right)^3\cdot x^4=x^{3a-3}\cdot x^4=x^{3a+1}.$

17b $\frac{1}{x^{1,53}}=x^{-1,53}.$

17d $\left(x^3\right)^a\cdot\sqrt{x}=x^{3a}\cdot x^{\frac{1}{2}}=x^{3a+\frac{1}{2}}.$

17f $\frac{x^5}{x^{a-1}}=x^{5-(a-1)}=x^{5-a+1}=x^{6-a}.$

18a $y=5x^4\cdot\sqrt{x}=5x^4\cdot x^{\frac{1}{2}}=5x^{4\frac{1}{2}}.$

18c $y=(3x^{-1,8})^4\cdot2x^{3,6}=81x^{-7,2}\cdot2x^{3,6}=162x^{-3,6}.$

```
3^4
               81
```

18b $y=\frac{5}{x}\cdot x^{1,3}=5x^{-1}\cdot x^{1,3}=5x^{0,3}.$

19a $y=18\cdot(2x^2)^{0,3}=18\cdot2^{0,3}\cdot x^{0,6}\approx22{,}16x^{0,6}.$

```
18*2^0.3
      22.16059944
```

19b $A=8\cdot\sqrt{3x}=8\cdot(3x)^{\frac{1}{2}}=8\cdot3^{\frac{1}{2}}\cdot x^{\frac{1}{2}}\approx13{,}86\cdot x^{0,5}.$

```
8*3^(1/2)
      13.85640646
```

19c $p=0{,}4x^3$ invullen in $Q=10\cdot p^3+\frac{8}{p}\Rightarrow Q=10\cdot\left(0{,}4x^3\right)^3+\frac{8}{0{,}4x^3}=10\cdot0{,}4^3\cdot x^9+\frac{8}{0{,}4}\cdot x^{-3}=0{,}64x^9+20x^{-3}.$

20a $x^5=18\Rightarrow x=\sqrt[5]{18}.$

```
5*√18
      1.782602458
18^(1/5)
      1.782602458
```

20b $\sqrt[3]{x}=4\Rightarrow x=4^3=64.$

```
4^3
               64
```

21a $y = 3x^{2,6} \Rightarrow x^{2,6} = \frac{1}{3}y \Rightarrow x = \left(\frac{1}{3}y\right)^{\frac{1}{2,6}} = \left(\frac{1}{3}\right)^{\frac{1}{2,6}} \cdot y^{\frac{1}{2,6}} \approx 0,66 \cdot y^{0,38}$.

```
(1/3)^(1/2.6)
      .6553779493
1/2.6
      .3846153846
```

21b $y = 0,18 \cdot x^{-1,4} \Rightarrow x^{-1,4} = \frac{1}{0,18}y \Rightarrow x = \left(\frac{1}{0,18}y\right)^{\frac{1}{-1,4}} \Rightarrow x = \left(\frac{1}{0,18}\right)^{\frac{1}{-1,4}} \cdot y^{\frac{1}{-1,4}} \approx 0,29 \cdot y^{-0,71}$.

```
(1/0.18)^(1/-1.4
)
       .2937999986
1/-1.4
      -.7142857143
```

21c $y = 2 \cdot \sqrt[3]{x} \Rightarrow \sqrt[3]{x} = \frac{1}{2}y \Rightarrow x = \left(\frac{1}{2}y\right)^3 \Rightarrow x = \left(\frac{1}{2}\right)^3 \cdot y^3 = \frac{1}{8}y^3$.

21d $y = 18 \cdot (3x)^{1,7} \Rightarrow (3x)^{1,7} = \frac{1}{18}y \Rightarrow 3x = \left(\frac{1}{18}y\right)^{\frac{1}{1,7}} \Rightarrow x = \frac{1}{3} \cdot \left(\frac{1}{18}\right)^{\frac{1}{1,7}} \cdot y^{\frac{1}{1,7}} \approx 0,06 \cdot y^{0,59}$.

```
1/3*(1/18)^(1/1.
7)
        .060881193
1/1.7
       .5882352941
```

22a $P = 2,5q^4 \Rightarrow q^4 = \frac{1}{2,5}P \Rightarrow q = \left(\frac{1}{2,5}P\right)^{\frac{1}{4}} = \left(\frac{1}{2,5}\right)^{\frac{1}{4}} \cdot P^{\frac{1}{4}} \approx 0,80 \cdot P^{0,25}$.

```
(1/2.5)^(1/4)
      .7952707288
1/4
              .25
```

22b $L = \frac{1}{6} \cdot \sqrt[3]{A} - 2 \Rightarrow \frac{1}{6} \cdot \sqrt[3]{A} = L + 2 \Rightarrow \sqrt[3]{A} = 6L + 12 \Rightarrow A = (6L + 12)^3$.

22c $a = 2P^{1,5}$ in $K = 4a^3$ geeft $K = 4 \cdot \left(2P^{1,5}\right)^3 = 4 \cdot (2)^3 \cdot P^{4,5} = 4 \cdot 8 \cdot P^{4,5} = 32 \cdot P^{4,5}$.

23a $100\% + 50\% = 150\%$, dus vermenigvuldigen met 1,5.

23b 1980 $N = 400 \cdot 1,5 \cdot 1,5 = 400 \cdot 1,5^2 = 900$.
1990 $N = 400 \cdot 1,5^3 = 1350$.
2000 $N = 400 \cdot 1,5^4 = 2025$.

```
400*1.5²
               900
400*1.5^3
              1350
400*1.5^4
              2025
```

23c Dit klopt niet, want 900 is meer dan het dubbel van 400 en 2025 is meer dan het dubbel va 900.

24a $N = 13,4 \cdot 1,029^t$.

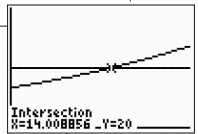
```
13.4*1.029^8
       16.84332375
```

24b Op 1-1-2012 is $t = 8 \Rightarrow N = 13,4 \cdot 1,029^8 \approx 16,8$ (miljoen inwoners).

24c $N = 13,4 \cdot 1,029^t = 20$ (intersect/TABLE) $\Rightarrow t = 14,...$ Dit is in $2004 + 14 = 2018$.

```
Plot1 Plot2 Plot3
\Y1=13.4*1.029^X
\Y2=20
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=0
 Ymax=40
 Yscl=0
↓Xres=1
```



24d Op 1-1-2014 is $t = 10 \Rightarrow N(10) \approx 17,83$ (miljoen inwoners).
Op 1-1-2015 is $t = 11 \Rightarrow N(11) \approx 18,35$ (miljoen inwoners).
De toename is $N(11) - N(10) \approx 0,52$ (miljoen inwoners).

```
Y1(10)
       17.83440176
Y1(11)
       18.35159942
Y1(11)-Y1(10)
       .5171976512
```

24e $N = 13,4 \cdot 1,029^t = 2 \cdot 13,4 = 26,8$ (intersect/TABLE) $\Rightarrow t = 24,...$ Dus in $2004 + 24 = 2028$.

X	Y1	Y2
21	24.425	20
22	25.133	20
23	25.862	20
24	26.612	20
25	27.384	20
26	28.178	20
27	28.995	20

X=25

25a $A = 42\,000 \cdot 1,08^t$.

25b Op 1-7-2016 is $t = 13\frac{1}{2} \Rightarrow A = 42\,000 \cdot 1,08^{13,5} \approx 119\,000$ (ha).

```
42000*1.08^13.5
       118705.2671
```

25c $A = 42\,000 \cdot 1,08^t = \frac{1}{4} \cdot 2\,000\,000 \Rightarrow t = 32,...$ Dus in $2003 + 32 = 2035$.

```
Plot1 Plot2 Plot3
\Y1=42000*1.08^X
\Y2=1/4*2000000
\Y3=
```

X	Y1	Y2
29	39133	500000
30	42263	500000
31	45644	500000
32	49296	500000
33	53239	500000
34	57499	500000
35	62098	500000

Y1=49295.7485807

26a $N_L = 700 \cdot 1,07^t$.

26b $N_K = 45 + 6t$.

```
Plot1 Plot2 Plot3
\Y1=700*1.07^X
\Y2=45+6X
\Y3=
```

26c Op 1-1-2000 is $N_L = 700 \cdot 1,07^5 \approx 982$.
Op 1-1-2001 is $N_L = 700 \cdot 1,07^6 \approx 1051$.
De toename in 2000 is $1051 - 982 = 69$. Dat is $\frac{69}{982} \times 100\% \approx 7\%$.
De toename in 2006 is 7%. (het aantal broedparen van de Lepelaar nam 7% per jaar toe)

```
700*1.07^5
       981.7862115
700*1.07^6
       1050.511246
(1051-982)/982*1
00
       7.026476578
```

```
Y1(5)
       981.7862115
Y1(6)
       1050.511246
Y1(6)/Y1(5)*100-
100
                 7
```

```
Y1(12)/Y1(11)*10
0-100
                 7
```

26d Op 1-1-2000 is $N_K = 45 + 6 \cdot 5 = 75$ en op 1-1-2001 is $N_K = 45 + 6 \cdot 6 = 81$.
De toename in 2000 is $81 - 75 = 6$. Dat is $\frac{6}{75} \times 100\% = 8\%$.
Op 1-1-2006 is $N_K = 45 + 6 \cdot 11 = 111$. De toename is elk jaar 6. Dus in 2006 $\frac{6}{111} \times 100\% \approx 5,4\%$.

```
Y2(5)
                75
Y2(6)
                81
81/75*100-100
                 8
```

```
Y2(11)
               111
6/111*100
       5.405405405
```

26e Bij de broedparen van de lepelaar is de toename in procenten elk jaar 7%.
Bij de broedparen van de grijze kiekendief is de toename van het aantal broedparen elk jaar 6 stuks.

27 $N = 5 \cdot 2^{t+3} = 5 \cdot 2^t \cdot 2^3 = 5 \cdot 2^3 \cdot 2^t = 5 \cdot 8 \cdot 2^t = 40 \cdot 2^t$.

28a $N = 25 \cdot 1,4^{3t-2} = 25 \cdot 1,4^{3t} \cdot 1,4^{-2} = 25 \cdot 1,4^{-2} \cdot (1,4^3)^t \approx 12,76 \cdot 2,74^t$.

```
25*1.4^-2
       12.75510204
1.4^3
             2.744
```

28b $N = 180 \cdot 0,8^{5-t} = 180 \cdot 0,8^5 \cdot 0,8^{-t} = 180 \cdot 0,8^5 \cdot (0,8^{-1})^t \approx 58,98 \cdot 1,25^t$.

```
180*0.8^5
           58.9824
0.8^-1
              1.25
```

28c $N = 40 \cdot 16^{2t-1} = 40 \cdot 16^{2t} \cdot 16^{-1} = 40 \cdot 16^{-1} \cdot (16^2)^t = 2,5 \cdot 256^t$.

```
40*16^-1
               2.5
16²
               256
```

29a $N = 200 \cdot 0{,}1^{0{,}02t} = 200 \cdot (0{,}1^{0{,}02})^t \approx 200 \cdot 0{,}95^t$.

```
0.1^0.02
      .954992586
```

29b $N = 1250 \cdot 0{,}8^{0{,}5t-1} = 1250 \cdot 0{,}8^{0{,}5t} \cdot 0{,}8^{-1} = 1250 \cdot 0{,}8^{-1} \cdot (0{,}8^{0{,}5})^t \approx 1562{,}5 \cdot 0{,}89^t$.

```
1250*0.8^-1
          1562.5
0.8^0.5
      .894427191
```

29c $N = 4 \cdot g^5 \Rightarrow g^5 = \frac{1}{4}N \Rightarrow g = (\frac{1}{4}N)^{\frac{1}{5}} \Rightarrow g = (\frac{1}{4})^{\frac{1}{5}} \cdot N^{\frac{1}{5}} \approx 0{,}76 \cdot N^{0{,}2}$.

```
(1/4)^(1/5)
     .7578582833
1/5
              .2
```

30a $x = 20 \cdot 1{,}6^{3t}$ invullen in $N = 200 - 2x \Rightarrow N = 200 - 2 \cdot 20 \cdot 1{,}6^{3t} = 200 - 40 \cdot (1{,}6^3)^t \approx 200 - 40 \cdot 4{,}1^t$.

```
1.6^3
           4.096
```

30b $x = 3 - 0{,}6x$ invullen in $N = 800 - 5 \cdot 1{,}8^t \Rightarrow$
$N = 800 - 5 \cdot 1{,}8^{3-0{,}6x} = 800 - 5 \cdot (1{,}8^3) \cdot (1{,}8^{-0{,}6x}) = 800 - 5 \cdot (1{,}8^3) \cdot (1{,}8^{-0{,}6})^x \approx 800 - 29 \cdot 0{,}7^x$.

```
5*1.8^3
           29.16
1.8^-0.6
     .7028076772
```

31a Met $2 \cdot 2 \cdot 2 = 2^3 = 8$. 31b Met $2^5 = 32$.

32a $g_{\text{jaar}} = 0{,}81 \Rightarrow g_{\text{maand}} = 0{,}81^{\frac{1}{12}} \approx 0{,}983 = 98{,}3\%$. De afname per maand is 1,7%.

```
(100-19)/100
             .81
Ans^(1/12)
     .9825931939
Ans*100-100
    -1.740680615
```

32b $g_{\text{week}} = 1{,}658 \Rightarrow g_{\text{dag}} = 1{,}658^{\frac{1}{7}} \approx 1{,}075 = 107{,}5\%$. De toename per dag is 7,5%.

```
1.658^(1/7)
     1.074902859
Ans*100-100
     7.490285904
```

32c $g_{\text{kwartier}} = 1{,}20 \Rightarrow g_{\text{uur}} = 1{,}20^4 \approx 2{,}074 = 207{,}4\%$. De toename per uur is 107,4%.

```
1.20^4
          2.0736
Ans*100-100
          107.36
```

32d $g_{\text{minuut}} = 0{,}9982 \Rightarrow g_{\text{uur}} = 0{,}9982^{60} \approx 0{,}898 = 89{,}8\%$. De afname per uur is 10,2%.

```
0.9982^60
     .8975402464
Ans*100-100
    -10.24597536
```

33a $g_{\text{week}} = 2{,}8 \Rightarrow g_{\text{dag}} = 2{,}8^{\frac{1}{7}} \approx 1{,}158$.

```
2.8^(1/7)
     1.158456468
```

33b $g_{\text{week}} = 2{,}8 \Rightarrow g_{\text{dag}} = 2{,}8^{\frac{1}{7 \cdot 24}} \approx 1{,}006 = 100{,}6\%$. Het groeipercentage per uur is 0,6%.

```
2.8^(1/(7*24))
     1.006147506
Ans*100-100
     .6147505835
```

34a $g_{5\,\text{jaar}} = \frac{210}{150} = 1{,}4 \Rightarrow g_{\text{jaar}} = 1{,}4^{\frac{1}{5}} \approx 1{,}07$.

34b $g_{\text{jaar}} = 1{,}4^{\frac{1}{5}} \approx 1{,}070 = 107{,}0\%$. De toename per jaar is 7,0%.

```
210/150
             1.4
Ans^(1/5)
     1.069610376
Ans*100-100
     6.961037573
```

35a $g_{25\,\text{jaar}} = \frac{20000}{80000} = 0{,}25 \Rightarrow g_{\text{jaar}} = 0{,}25^{\frac{1}{25}} \approx 0{,}946 = 94{,}6\%$. De afname per jaar is 5,4%.

```
20000/80000
             .25
Ans^(1/25)
     .9460576467
Ans*100-100
    -5.394235327
```

35b $g_{10\,\text{jaar}} = 8 \Rightarrow g_{\text{jaar}} = 8^{\frac{1}{10}} \approx 1{,}231 = 123{,}1\%$. Het groeipercentage per jaar is 23,1%.

```
8^0.1
     1.231144413
Ans*100-100
     23.11444133
```

35c $g_{15\,\text{jaar}} = 0{,}5 \Rightarrow g_{\text{jaar}} = 0{,}5^{\frac{1}{15}} \approx 0{,}955 = 95{,}5\%$. De afname per jaar is 4,5%.

```
0.5^(1/15)
     .9548416039
Ans*100-100
    -4.515839609
```

36a $g_{45\,\text{jaar}} = 0{,}70 \Rightarrow g_{\text{jaar}} = 0{,}70^{\frac{1}{45}} \approx 0{,}992 = 99{,}2\%$. De afname per jaar is 0,8%.

```
0.7^(1/45)
     .9921052189
Ans*100-100
    -.7894781083
```

36b $g_{10\,\text{jaar}} = \frac{1500}{30000} = 0{,}05 \Rightarrow g_{\text{jaar}} = 0{,}05^{\frac{1}{10}} \approx 0{,}741 = 74{,}1\%$. De afname per jaar is 25,9%.

```
1500/30000
             .05
Ans^0.1
     .7411344491
Ans*100-100
    -25.88655509
```

36c $g_{4\,\text{jaar}} = \frac{17000}{12000} = \frac{17}{12} \Rightarrow g_{\text{jaar}} = \left(\frac{17}{12}\right)^{\frac{1}{4}} \approx 1{,}091 = 109{,}1\%$. De toename per jaar is 9,1%.

```
(17/12)^(1/4)
     1.090980326
Ans*100-100
     9.098032586
```

36d $g_{12\,\text{jaar}} = 1{,}38 \Rightarrow g_{\text{jaar}} = 1{,}38^{\frac{1}{12}} \approx 1{,}027 = 102{,}7\%$. De toename per jaar is 2,7%.

```
1.38^(1/12)
     1.027203737
Ans*100-100
     2.720373659
```

36e $g_{15\,\text{jaar}} = \frac{7{,}4}{6{,}4} \Rightarrow g_{\text{jaar}} = \left(\frac{7{,}4}{6{,}4}\right)^{\frac{1}{15}} \approx 1{,}0097$.

```
(7.4/6.4)^(1/15)
     1.009725792
```

37a $g_{7\,\text{jaar}} = \frac{135}{117} \Rightarrow g_{\text{jaar}} = \left(\frac{135}{117}\right)^{\frac{1}{7}} \approx 1{,}021$.

```
(135/117)^(1/7)
     1.020653367
```

37b $N = 117 \cdot 1{,}021^t$.

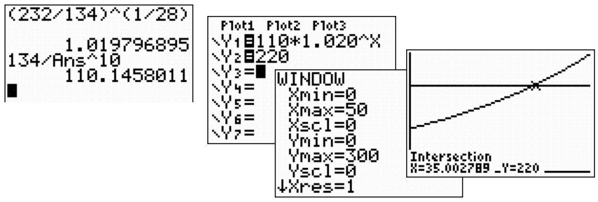
38 $g_{6\,\text{uur}} = \frac{1250000}{150000} = \frac{25}{3} \Rightarrow g_{\text{uur}} = \left(\frac{25}{3}\right)^{\frac{1}{6}} = 1{,}42...$
$N = b \cdot 1{,}42...^t$ en bij $t = 2$ hoort $N = 150\,000 \Rightarrow 150\,000 = b \cdot 1{,}42...^2 \Rightarrow b\frac{150000}{1{,}42...^2} \approx 74\,000$. Dus $N = 74\,000 \cdot 1{,}42^t$.

```
1250000/150000►F
rac
            25/3
Ans^(1/6)
     1.423868205
150000/Ans²
     73986.36223
```

39a $N = b \cdot g^t$ met $g^{28} = \frac{232}{134} \Rightarrow g = \left(\frac{232}{134}\right)^{\frac{1}{28}} = 1{,}0197...$

$\left.\begin{array}{l} N = b \cdot 1{,}0197...^t \\ t = 10 \Rightarrow N = 134 \end{array}\right\} \Rightarrow 134 = b \cdot 1{,}0197...^{10} \Rightarrow b = \frac{134}{1{,}0197...^{10}} \approx 110.$

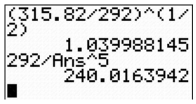
Dus $N \approx 110 \cdot 1{,}020^t$.



39b $N = 110 \cdot 1{,}020^t = 220$ (intersect) $\Rightarrow t \approx 35$. Dus in 35 jaar.

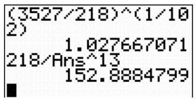
40 $N = b \cdot g^t$ met $g^2 = \frac{315{,}82}{292} \Rightarrow g = \left(\frac{315{,}82}{292}\right)^{\frac{1}{2}} = 1{,}039...$

$\left.\begin{array}{l} N = b \cdot 1{,}039...^t \\ t = 5 \Rightarrow N = 292 \end{array}\right\} \Rightarrow 292 = b \cdot 1{,}039...^5 \Rightarrow b = \frac{292}{1{,}039...^5} \approx 240.$ Dus Rob heeft € 240 op de bank gezet.

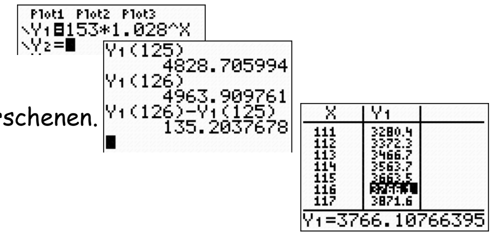


41a $C = b \cdot g^t$ met $g^{102} = \frac{3527}{218} \Rightarrow g = \left(\frac{3527}{218}\right)^{\frac{1}{102}} = 1{,}027...$

$\left.\begin{array}{l} C = b \cdot 1{,}027...^t \\ t = 13 \Rightarrow C = 218 \end{array}\right\} \Rightarrow 218 = b \cdot 1{,}027...^{13} \Rightarrow b = \frac{218}{1{,}027...^{13}} \approx 153.$ Dus $C \approx 153 \cdot 1{,}028^t$.



41b Op 1-1-2000 is $t = 125 \Rightarrow C = 153 \cdot 1{,}028^{125} \approx 4\,829$.
Op 1-1-2000 is $t = 126 \Rightarrow C = 153 \cdot 1{,}028^{126} \approx 4\,964$.
In het jaar zijn er (volgens dit model) $4\,964 - 4\,829 = 135$ postzegels verschenen.



41c $C = 153 \cdot 1{,}028^t > 100$ (TABLE) $\Rightarrow$ $t = 114 \Rightarrow C \approx 3563{,}7$
$t = 115 \Rightarrow C \approx 3663{,}5$
$t = 116 \Rightarrow C \approx 3766{,}1.$

Van $t = 115$ tot $t = 116$ is er voor het eerst meer dan 100 bij gekomen. Bij $t = 115$ hoort 1-1-1990. Dus in 1990.

42a $\frac{\Delta N}{\Delta t} = \frac{90-50}{12-8} = \frac{40}{4} = 10$ (dit is de toename van N per tijdseenheid).

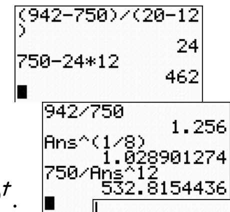
42b $g^{12-8} = \frac{90}{50} \Rightarrow g^4 = \frac{9}{5} = 1{,}8 \Rightarrow g = 1{,}8^{\frac{1}{4}} \approx 1{,}16$ (dit is de groeifactor per tijdseenheid).

43a $N_1 = at + b$ met $a = \frac{\Delta N}{\Delta t} = \frac{942-750}{20-12} = \frac{192}{8} = 24.$

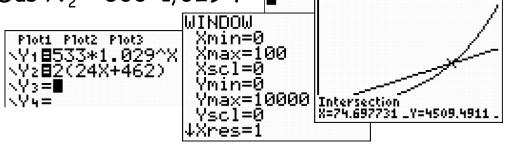
$\left.\begin{array}{l} N_1 = 24t + b \\ t = 12 \Rightarrow N_1 = 750 \end{array}\right\} \Rightarrow 750 = 24 \cdot 12 + b \Rightarrow b = 750 - 24 \cdot 12 = 462.$ Dus $N_1 = 24t + 462$.

$N_2 = b \cdot g^t$ met $g^{20-12} = g^8 = \frac{942}{750} = 1{,}256 \Rightarrow g = 1{,}256^{\frac{1}{8}} = 1{,}0289...$

$\left.\begin{array}{l} N_2 = b \cdot 1{,}0289...^t \\ t = 12 \Rightarrow N_2 = 750 \end{array}\right\} \Rightarrow 750 = b \cdot 1{,}0289...^{12} \Rightarrow b = \frac{750}{1{,}0289...^{12}} \approx 533.$ Dus $N_2 \approx 533 \cdot 1{,}029^t$.



43b $N_2 = 2N_1 \Rightarrow 533 \cdot 1{,}029^t = 2(24t + 462)$ (intersect) $\Rightarrow t \approx 74{,}7$.

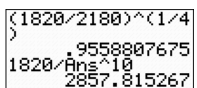


44a $N_1 = at + b$ met $a = \frac{\Delta N}{\Delta t} = \frac{1820-2180}{10-6} = \frac{-360}{4} = -90.$

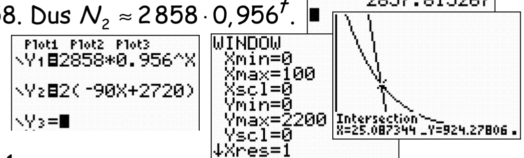
$\left.\begin{array}{l} N_1 = -90t + b \\ t = 10 \Rightarrow N_1 = 1820 \end{array}\right\} \Rightarrow 1820 = -90 \cdot 10 + b \Rightarrow b = 1820 + 900 = 2\,720.$ Dus $N_1 = -90t + 2\,720$.

$N_2 = b \cdot g^t$ met $g^{10-6} = g^4 = \frac{1820}{2180} \Rightarrow g = \left(\frac{1820}{2180}\right)^{\frac{1}{4}} = 0{,}9558...$

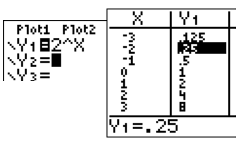
$\left.\begin{array}{l} N_2 = b \cdot 0{,}9558...^t \\ t = 10 \Rightarrow N_2 = 1820 \end{array}\right\} \Rightarrow 1820 = b \cdot 0{,}9558...^{10} \Rightarrow b = \frac{1820}{0{,}9558...^{10}} \approx 2\,858.$ Dus $N_2 \approx 2\,858 \cdot 0{,}956^t$.



44b $N_2 = 2N_1 \Rightarrow 2\,858 \cdot 0{,}956^t = 2(-90t + 2\,720)$ (intersect) $\Rightarrow t \approx 25{,}1$.



45 $f(3) = 2^3 = 8, \quad 16 = 2^4 = f(4), \quad \frac{1}{2} = 2^{-1} = f(-1), \quad \sqrt{2} = 2^{\frac{1}{2}} = f(\frac{1}{2}).$



46a $f^{\text{inv}}(16) = 4$, want $2^4 = 16$.

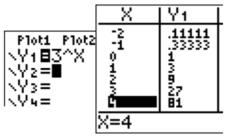
46b $f^{\text{inv}}(4) = 2$, want $2^2 = 4$.

46c $f^{\text{inv}}(1) = 0$, want $2^0 = 1$.

46d $f^{\text{inv}}(\frac{1}{2}) = -1$, want $2^{-1} = \frac{1}{2}$.

47a $g^{\text{inv}}(9) = 2$, want $3^2 = 9$.

47b $g^{\text{inv}}(81) = 4$, want $3^4 = 81$.



47c $g^{\text{inv}}(1) = 0$, want $3^0 = 1$.

47d $g^{\text{inv}}(\frac{1}{3}) = -1$, want $3^{-1} = \frac{1}{3}$.

48a $^{2}\log(32) = 5$, want $2^5 = 32$.

48b $^{3}\log(\frac{1}{3}) = -1$, want $3^{-1} = \frac{1}{3}$.

48c $^{5}\log(25) = 2$, want $5^2 = 25$.

48d $^{6}\log(\sqrt{6}) = \frac{1}{2}$, want $6^{\frac{1}{2}} = \sqrt{6}$.

$^{g}\log(x) = y$ betekent $g^y = x$
dus $^{g}\log(g^y) = y$

49a $^{5}\log(125) = {}^{5}\log(5^3) = 3$.

49b $^{10}\log(\frac{1}{10}) = {}^{10}\log(10^{-1}) = -1$.

49c $^{2}\log(4) = {}^{2}\log(2^2) = 2$.

49d $^{7}\log(49) = {}^{7}\log(7^2) = 2$.

49e $^{2}\log(\sqrt{2}) = {}^{2}\log(2^{\frac{1}{2}}) = \frac{1}{2}$.

49f $^{3}\log(27) = {}^{3}\log(3^3) = 3$.

49g $^{2}\log(\frac{1}{16}) = {}^{2}\log(\frac{1}{2^4}) = {}^{2}\log(2^{-4}) = -4$.

49h $^{4}\log(\frac{1}{4}) = {}^{4}\log(4^{-1}) = -1$.

49i $^{5}\log(5) = {}^{5}\log(5^1) = 1$.

49j $^{6}\log(1) = {}^{6}\log(6^0) = 0$.

49k $^{7}\log(\sqrt{7}) = {}^{7}\log(7^{\frac{1}{2}}) = \frac{1}{2}$.

49l $^{2}\log(\frac{1}{4}) = {}^{2}\log(\frac{1}{2^2}) = {}^{2}\log(2^{-2}) = -2$.

50a Omdat $2^{-2} = \frac{1}{2^2} = \frac{1}{4} \neq -4$.

50b Omdat $1^8 = 1 \neq 8$.

50c $1^8 = 1$, maar ook $1^7 = 1$; dan zou ook $^{1}\log(1) = 7$. Dit kan natuurlijk niet.

51a $^{2}\log(16 \cdot \sqrt{2}) = {}^{2}\log(2^4 \cdot 2^{\frac{1}{2}}) = {}^{2}\log(2^{4\frac{1}{2}}) = 4\frac{1}{2}$.

51b $^{3}\log(\frac{1}{9}) = {}^{3}\log(3^{-2}) = -2$.

51c $^{3}\log(3^{2,76}) = 2,76$.

51d $^{5}\log(\frac{1}{125}) = {}^{5}\log(5^{-3}) = -3$.

51e $^{4}\log(\frac{1}{4}) = {}^{4}\log(4^{-1}) = -1$.

51f $^{10}\log(10) = {}^{10}\log(10^1) = 1$.

51g $^{2}\log(\frac{1}{32}) = {}^{2}\log(2^{-5}) = -5$.

51h $^{4}\log(1) = {}^{4}\log(4^0) = 0$.

51i $^{3}\log(81) = {}^{3}\log(3^4) = 4$.

51j $^{5}\log(5^{-6\frac{1}{2}}) = -6\frac{1}{2}$.

51k $^{10}\log(0,1) = {}^{10}\log(\frac{1}{10}) = {}^{10}\log(10^{-1}) = -1$.

51l $^{10}\log(10\,000) = {}^{10}\log(10^4) = 4$.

$g^{\cdots}$ en $^{g}\log(\ldots)$ heffen elkaar op

52 1) $f(\frac{1}{8}) = {}^{2}\log(\frac{1}{8}) = {}^{2}\log(\frac{1}{2^3}) = {}^{2}\log(2^{-3}) = -3$.

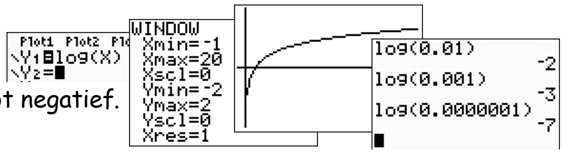
2) $f(4\sqrt{2}) = {}^{2}\log(4 \cdot \sqrt{2}) = {}^{2}\log(2^2 \cdot 2^{\frac{1}{2}}) = {}^{2}\log(2^{2\frac{1}{2}}) = 2\frac{1}{2}$.

3) $f(\sqrt[5]{4}) = {}^{2}\log(\sqrt[5]{4}) = {}^{2}\log(\sqrt[5]{2^2}) = {}^{2}\log(2^{\frac{2}{5}}) = \frac{2}{5}$.

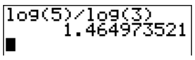
4) $f(1) = {}^{2}\log(1) = {}^{2}\log(2^0) = 0$.

53a Zie de plot hiernaast.

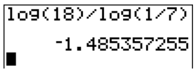
53bc $f(0,01) = -2$; $f(0,001) = -3$; $f(0,000\,0001) = -7$.
Als je x steeds dichter bij nul kiest, dan wordt $f(x)$ heel groot negatief.
De y-as is de verticale asymptoot van de grafiek van f.



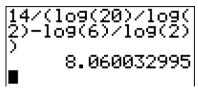
54a $^{3}\log(5) = \frac{\log(5)}{\log(3)} \approx 1,46$.



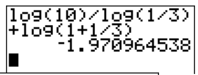
54b $^{\frac{1}{7}}\log(18) = \frac{\log(18)}{\log(\frac{1}{7})} \approx -1,49$.



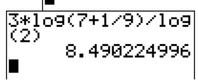
54c $\frac{14}{^{2}\log(20) - {}^{2}\log(6)} = \frac{14}{\frac{\log(20)}{\log(2)} - \frac{\log(6)}{\log(2)}} \approx 8,06$.



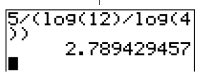
54d $^{\frac{1}{3}}\log(10) + \log(1\frac{1}{3}) = \frac{\log(10)}{\log(\frac{1}{3})} + \log(1\frac{1}{3}) \approx -1,97$.



54e $3 \cdot {}^{2}\log(7\frac{1}{9}) = 3 \cdot \frac{\log(7\frac{1}{9})}{\log(2)} \approx 8,49$.



54f $\frac{5}{^{4}\log(12)} = \frac{5}{\frac{\log(12)}{\log(4)}} \approx 2,79$.



55a $y = {}^{3}\log(x)$
↓ .....translatie (5, 0)
$y = {}^{3}\log(x - 5)$.

55b $y = {}^{3}\log(x)$
↓ .....translatie (0, 3)
$y = {}^{3}\log(x) + 3$.

55c $y = {}^{3}\log(x)$
↓ .....translatie (−3, 0)
$y = {}^{3}\log(x + 3)$.

55d $y = {}^{3}\log(x)$
↓ .....translatie (4, −3)
$y = {}^{3}\log(x - 4) - 3$.

55e $y = {}^{3}\log(x)$
↓ .....verm. t.o.v. de x-as met 3
$y = 3 \cdot {}^{3}\log(x)$.

55f $y = {}^{3}\log(x)$
↓ .....spiegeling in de x-as
$y = -{}^{3}\log(x)$.

56a $y = \log(x)$ met D(omein) $= \langle 0, \rightarrow \rangle$ en V(ert.) A(symptoot): $x = 0$.
↓ .....translatie (−6, 0)
$y = \log(x + 6)$ met D $= \langle -6, \rightarrow \rangle$ en V.A.: $x = -6$.

56b $y = \log(x)$ met D $= \langle 0, \rightarrow \rangle$ en V.A.: $x = 0$.
↓ .....translatie (0, 6)
$y = \log(x) + 6$ met D $= \langle 0, \rightarrow \rangle$ en V.A.: $x = 0$.

56c $y = \log(x)$ met $D = \langle 0, \rightarrow\rangle$ en V.A.: $x = 0$.
$\Downarrow$ .....translatie (4, 5)
$y = \log(x-4)+5$ met $D = \langle 4, \rightarrow\rangle$ en V.A.: $x = 4$.

56d $y = \log(x)$ met $D = \langle 0, \rightarrow\rangle$ en V.A.: $x = 0$.
$\Downarrow$ .....vermenigvuldiging t.o.v. de x-as met -3
$y = -3\cdot\log(x)$ met $D = \langle 0, \rightarrow\rangle$ en V.A.: $x = 0$.

57a $x-5=0 \Rightarrow x=5 \Rightarrow$ V.A.: $x=5$ en $D=\langle 5, \rightarrow\rangle$.

57b $x=0 \Rightarrow$ V.A.: $x=0$ en $D=\langle 0, \rightarrow\rangle$.

57c $x+5=0 \Rightarrow x=-5 \Rightarrow$ V.A.: $x=-5$ en $D=\langle -5, \rightarrow\rangle$.

57d $x+1=0 \Rightarrow x=-1 \Rightarrow$ V.A.: $x=-1$ en $D=\langle -1, \rightarrow\rangle$.

57e $x=0 \Rightarrow$ V.A.: $x=0$ en $D=\langle 0, \rightarrow\rangle$.

57f $x=0 \Rightarrow$ V.A.: $x=0$ en $D=\langle 0, \rightarrow\rangle$.

58a $\log(1000)=3$, want $10^3 = 1000$ of $\log(1000)=\log(10^3)=3$.

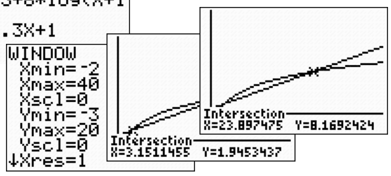
58b $\log(0{,}01)=-2$, want $10^{-2}=\frac{1}{10^2}=\frac{1}{100}=0{,}01$ of $\log(0{,}01)=\log(\frac{1}{100})=\log(10^{-2})=-2$.

59a $f(5)\approx 3{,}23$.
$f(250)\approx 16{,}20$.
$f(1000)\approx 21{,}00$.

```
-3+8*log(6)
        3.225210003
-3+8*log(251)
        16.19738977
-3+8*log(1001)
        21.00347262
```

```
Plot1 Plot2 Plot3
\Y1=-3+8*log(X+1)
\Y2=0.3X+1
\Y3=
```

59c $f(x)=g(x)$ (voorwaarde $x+1>0 \Rightarrow x>-1$)
Intersect geeft $x\approx 3{,}15 \vee x\approx 23{,}90$.
$f(x)=g(x)$ (zie plot) $\Rightarrow 3{,}15 < x < 23{,}90$.



59b $-3+8\cdot\log(x+1)=17$
$8\cdot\log(x+1)=20$
$\log(x+1)=2{,}5$
$x+1=10^{2{,}5}$
$x=10^{2{,}5}-1\approx 315{,}23$.

```
20/8
                2.5
10^2.5
        316.227766
Ans-1
        315.227766
```

60a $3+\log(x+20)=6$
$\log(x+20)=3$
$x+20=10^3=1000$
$x=980$.

60b $-2+5\cdot\log(x+9)=8$
$5\cdot\log(x+9)=10$
$\log(x+9)=2$
$x+9=10^2=100$
$x=91$.

60c $\log(4x+800)=3$
$4x+800=10^3=1000$
$4x=200$
$x=50$.

60d $2+5\cdot\log(x-8)=13$
$5\cdot\log(x-8)=11$
$\log(x-8)=2{,}2$
$x-8=10^{2{,}2}\approx 158{,}49$
$x\approx 166{,}49$.

```
10^2.2
        158.4893192
```

60e $8-4\cdot\log(x)=5$
$-4\cdot\log(x)=-3$
$\log(x)=0{,}75$
$x=10^{0{,}75}\approx 5{,}62$.

```
10^0.75
        5.623413252
```

60f $20\,000+8\,000\cdot\log(x+400)=60\,000$
$8\,000\cdot\log(x+400)=40\,000$
$\log(x+400)=5$
$x+400=10^5=100\,000$
$x=99\,600$.

61a $x=2\frac{3}{12}=2\frac{1}{4}\Rightarrow h=70{,}228+5{,}104\cdot 2{,}25+21{,}234\cdot\log(2{,}25)\approx 89$ (cm).

```
Plot1 Plot2 Plot3
\Y1=70.228+5.104
X+21.234*log(X)
\Y2=100
\Y3=
```

```
Y1(2.25)
        89.19024359
```

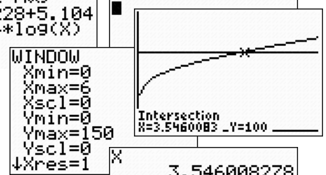
61b $\frac{h(5)-h(4)}{h(4)}\times 100\% \approx 6{,}9\%$.

```
(Y1(5)-Y1(4))/Y1
(4)*100
        6.924408665
```

61c Bij 1-12-2000 hoort $t=\frac{9}{12}$ en bij 1-1-2001 hoort $t=\frac{10}{12}$.
$h(\frac{10}{12})-h(\frac{9}{12})\approx 1{,}4 \Rightarrow$ hij is in december 14 mm gegroeid.

```
Y1(10/12)-Y1(9/1
2)
        1.396947888
```

61d $h=100$ (intersect) $\Rightarrow x\approx 3{,}546...$ (jaar). Dit is 3 jaar en (afgerond) 7 maanden.




```
X
        3.546008278
Ans-3
        .5460082782
Ans*12
        6.552099338
```

62a DIN $=1+k\cdot\log(\text{ISO})$ met 100 ISO = 21 DIN $\Rightarrow 21=1+k\cdot\log(100)\Rightarrow 20=k\cdot\log(10^2)\Rightarrow 20=k\cdot 2\Rightarrow k=\frac{20}{2}=10$.

62b 400 ISO/ASA $\Rightarrow$ din $=1+10\cdot\log(400)\approx 27$. Dus 27 DIN.

```
1+10log(400)
        27.02059991
```

```
log(100)
                  2
```

```
10^2.3
        199.5262315
```

62c 24 DIN $\Rightarrow 24=1+10\cdot\log(\text{ISO})\Rightarrow 23=10\cdot\log(\text{ISO})\Rightarrow 2{,}3=\log(\text{ISO})\Rightarrow$ iso $=10^{2{,}3}\approx 200$. Dus 200 ISO/ASA.

63a $d=4$ (cm) en $W=1{,}8$ (cm) $\Rightarrow T=75+20\cdot\log(\frac{4}{1{,}8}+1)\approx 85$ (ms).

```
75+20*log(4/1.8+
1)
        85.16310977
```

63b $W=1{,}5$ (cm) en $T=92$ (ms)
$75+20\cdot\log(\frac{d}{1{,}5}+1)=92$
$20\cdot\log(\frac{d}{1{,}5}+1)=17$
$\log(\frac{d}{1{,}5}+1)=\frac{17}{20}=0{,}85$
$\frac{d}{1{,}5}+1=10^{0{,}85}$
$\frac{d}{1{,}5}=10^{0{,}85}-1$
$d\approx 9$ (cm).

```
17/20
                .85
10^0.85-1
        6.079457844
Ans*1.5
        9.119186766
```

63c $T=80$ (ms) en $d=8$ (cm)
$75+20\cdot\log(\frac{8}{W}+1)=80$
$20\cdot\log(\frac{8}{W}+1)=5$
$\log(\frac{8}{W}+1)=\frac{1}{4}=0{,}25$
$\frac{8}{W}+1=10^{0{,}25}$
$\frac{8}{W}=10^{0{,}25}-1$
$W=\frac{8}{10^{0{,}25}-1}\approx 10{,}3$ (cm). Dus de breedte is 103 mm.

```
10^0.25
        1.77827941
Ans-1
        .77827941
8/Ans
        10.27908473
```

64a Ze betalen $V + 50 = 59{,}27 \cdot \log(240\,000) - 191{,}40 + 50 \approx 177{,}49$ (€).

```
59.27log(240000)
-191.40+50
        177.4851203
```

64b Ze betalen in maart $59{,}27 \cdot \log(214\,000) - 59{,}27 \cdot \log(158\,000) \approx 7{,}81$ (€) meer.

```
158000+56000
             214000
59.27log(214000)
-59.27log(158000
)
        7.809218803
```

64c $59{,}27 \cdot \log(n) - 191{,}40 = 0$
$59{,}27 \cdot \log(n) = 191{,}40$
$\log(n) = \frac{191{,}40}{59{,}27}$
$n = 10^{\frac{191{,}40}{59{,}27}} \approx 1700.$
De formule geldt vanaf $n = 1700$.

```
191.4/59.27
        3.229289691
10^Ans
        1695.468366
```

64d Vast bedrag per jaar is $12 \cdot 50 = 600$ (€).
Variabele kosten in dat jaar $2200 - 600 = 1600$ (€).
Dus $V = \frac{1600}{12} = 133{,}...$ (€).
$59{,}27 \cdot \log(n) - 191{,}40 = 133{,}...$
$59{,}27 \cdot \log(n) = 423{,}...$
$\log(n) = 5{,}47...$
$n = 10^{5{,}47...} \approx 301\,000$ (bezoekers / maand).

```
1600/12+191.40
        324.7333333
Ans/59.27
        5.478881953
10^Ans
        301218.7158
```

64e Bij verdubbeling van n stijgt V steeds met (ongeveer) € 17,84. (zie de getalvoorbeelden hiernaast)

```
Plot1 Plot2 Plot3
\Y1=59.27*log(X)
-191.40
\Y2=
```

```
Y1(2000)-Y1(1000
)
        17.84204784
Y1(20000)-Y1(100
00)
        17.84204784
```

```
Y1(80000)-Y1(400
00)
        17.84204784
Y1(24680)-Y1(123
40)
        17.84204784
```

65a 8 uur is $\frac{8 \cdot 60 \cdot 60}{15} = 1920$ keer 15 seconden. Dus maximaal 1920 te verdienen in 8 uur.

```
8*60*4
               1920
```

65b $B = 3228 \cdot \frac{2 \cdot \log(848)}{\sqrt{848}} \approx 649.$

```
3228*2*log(848)/
√(848)
        649.2252146
```

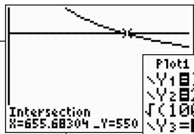
```
12*60*4
               2880
```

65c 12 uur is $\frac{12 \cdot 60 \cdot 60}{15} = 2880$ keer 15 seconden $\Rightarrow$ ze heeft 2880 punten verdiend en $12 \cdot 60 = 720 = d$ km gereden.
$B = 2880 \cdot \frac{4 \cdot \log(720)}{\sqrt{720}} \approx 1227.$

```
2880*4*log(720)/
√(720)
        1226.724422
```

```
Plot1 Plot2 Plot3
\Y1=2500*2log(X)
/√(X)
\Y2=550
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=1000
 Xscl=0
 Ymin=0
 Ymax=700
 Yscl=0
 Xres=1
```

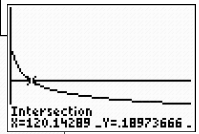


65d $550 = 2500 \cdot \frac{2 \cdot \log(d)}{\sqrt{d}}$ (intersect) $\Rightarrow d \approx 660$ (km).

65e $B_E = 2 \cdot B_F \Rightarrow \frac{\log(d)}{\sqrt{d}} = 2 \cdot \frac{\log(1000)}{\sqrt{1000}}$ of $\frac{\log(d)}{\sqrt{d}} = 2 \cdot \frac{3}{\sqrt{1000}}$.
Intersect geeft $d \approx 120$ (voor het aantal gereden km van Ellen).

```
Plot1 Plot2 Plot3
\Y1=log(X)/√(X)
\Y2=2*log(1000)/
√(1000)
\Y3=
\Y4=
\Y5=
\Y6=
```

```
WINDOW
 Xmin=0
 Xmax=1000
 Xscl=0
 Ymin=0
 Ymax=0.5
 Yscl=0
 Xres=1
```



65f Vergelijk iemand die 100 km per week rijdt met iemand die 400 km per week rijdt.
Neem aan dat voor beide rijders de gemiddelde snelheid 50 km per uur is en neem verder aan dat $K = 1$.
Bij 100 km per week is dan de tijd 2 uur en het aantal maximaal te verdienen punten $\frac{2 \cdot 60 \cdot 60}{15} = 2 \cdot 60 \cdot 4 = 480.$
Bij 400 km per week is de tijd 8 uur en het aantal maximaal te verdienen punten 1920 (zie ook 65a).
Bij 100 km per week is $B = 480 \cdot \frac{\log(100)}{10} = 96$ en bij 400 km per week is $B = 1920 \cdot \frac{\log(400)}{20} \approx 250$.
Dus bij 4 keer zoveel rijden per week in dit geval slechts $\frac{250}{96} \approx 2{,}6$ keer zoveel punten.

```
48log(100)
                 96
96log(400)
        249.7977592
```

```
250/96
        2.604166667
```

66a Bij een octaaf is $f_2 : f_1 = 2 : 1 = 2 \Rightarrow d = 1200 \cdot {}^2\log(2) = 1200 \cdot {}^2\log(2^1) = 1200 \cdot 1 = 1200$ cent.

66b Bij een grote sext is $f_2 : f_1 = 5 : 3 \Rightarrow d = 1200 \cdot {}^2\log(5:3) = 1200 \cdot {}^2\log(\frac{5}{3}) = 1200 \cdot \frac{\log(\frac{5}{3})}{\log(2)} \approx 880$ cent

```
1200*log(5/3)/lo
g(2)
        884.358713
```

66c $d = 1200 \cdot {}^2\log(f_2 : f_1) = 1200 \cdot \frac{\log(f_2 : f_1)}{\log(2)} = \frac{1200}{\log(2)} \cdot \log(f_2 : f_1) \approx 3986 \cdot \log(f_2 : f_1) \approx 4000 \cdot \log(f_2 : f_1).$

```
1200/log(2)
        3986.313714
```

66d $4000 \cdot \log(f_2 : f_1) = 500 \Rightarrow \log(f_2 : f_1) = \frac{1}{8} = 0{,}125 \Rightarrow f_2 : f_1 = 10^{0{,}125} \approx 1{,}33 \approx \frac{4}{3} = 4:3.$

```
10^0.125
        1.333521432
```

66e Bij een kwint is $d \approx 4000 \cdot \log(\frac{3}{2}) \approx 700.$
Bij een terts is $d \approx 4000 \cdot \log(\frac{5}{4}) \approx 390.$
Bij een septiem is $d \approx 4000 \cdot \log(\frac{15}{8}) \approx 1090.$

```
4000*log(3/2)
        704.3650362
4000*log(5/4)
        387.640052
4000*log(15/8)
        1092.005088
```

Er geldt $700 + 390 = 1090$.

67a $y = \log(x) + \log(5)$ en $y = \log(5x)$ komen op hetzelfde neer.

67b $y = \log(x) - \log(5)$ en $y = \log(\frac{x}{5})$ komen op hetzelfde neer.

67c $y = \log(x^3)$ en $y = 3 \cdot \log(x)$ komen op hetzelfde neer.

```
Plot1 Plot2 Plot3
\Y1=log(X+5)
\Y2=log(X)+log(5
)
\Y3=log(5X)
\Y4=
\Y5=
\Y6=
```

X	Y2	Y3
0	ERROR	ERROR
1	.69897	.69897
2	1	1
3	1.1761	1.1761
4	1.301	1.301
5	1.3979	1.3979
6	1.4771	1.4771

Y3=1.17609125906

```
Plot1 Plot2 Plot3
\Y1=log(X-5)
\Y2=log(X)-log(5
)
\Y3=log(X/5)
\Y4=
\Y5=
\Y6=
```

X	Y2	Y3
0	ERROR	ERROR
1	-.699	-.699
2	-.3979	-.3979
3	-.2218	-.2218
4	-.0969	-.0969
5	0	0
6	.07918	.07918

Y3=-.39794000867

68a $^2\log(7) + {}^2\log(6) = {}^2\log(7 \cdot 6) = {}^2\log(42).$

68b $^2\log(15) - {}^2\log(3) = {}^2\log(\frac{15}{3}) = {}^2\log(5).$

68c $2 \cdot {}^2\log(3) - 3 \cdot {}^2\log(5) = {}^2\log(3^2) - {}^2\log(5^3) = {}^2\log(9) - {}^2\log(125) = {}^2\log(\frac{9}{125}).$

```
Plot1 Plot2 Plot3
\Y1=log(X^3)
\Y2=3log(X)
\Y3=(log(X))^3
\Y4=
\Y5=
\Y6=
\Y7=
```

X	Y1	Y2
0	ERROR	ERROR
1	0	0
2	.90309	.90309
3	1.4314	1.4314
4	1.8062	1.8062
5	2.0969	2.0969
6	2.3345	2.3345

X=4

68d $3 + \log(5) = \log(10^3) + \log(5) = \log(1000) + \log(5) = \log(1000 \cdot 5) = \log(5000).$

68e $\log(5) + 3 \cdot \log(3) = \log(5) + \log(3^3) = \log(5) + \log(27) = \log(5 \cdot 27) = \log(135).$

68f $\log(50) - 2 \cdot \log(5) = \log(50) - \log(5^2) = \log(\frac{50}{25}) = \log(2).$

69a $^2\log(a)+3\cdot {}^2\log(b)={}^2\log(a)+{}^2\log(b^3)={}^2\log(ab^3)$.

69b $5\cdot {}^3\log(a)-2\cdot {}^3\log(b)={}^3\log(a^5)-{}^3\log(b^2)={}^3\log\left(\frac{a^5}{b^2}\right)$.

69c $2+{}^5\log(a)={}^5\log(5^2)+{}^5\log(a)={}^5\log(25)+{}^5\log(a)={}^5\log(25a)$.

69d $2-\log(a)=\log(10^2)-\log(a)=\log(100)-\log(a)=\log\left(\frac{100}{a}\right)$.

69e $\log(a)-1=\log(a)-\log(10^1)=\log(a)-\log(10)=\log\left(\frac{a}{10}\right)=\log\left(\frac{1}{10}a\right)$.

69f $2\cdot\log(b)+\frac{1}{2}\cdot\log(a)=\log(b^2)+\log(a^{\frac{1}{2}})=\log(b^2)+\log(\sqrt{a})=\log(b^2\cdot\sqrt{a})$.

70 De regel ${}^g\log(a)+{}^g\log(b)={}^g\log(ab)$. (namelijk $\log(\frac{3}{2})+\log(\frac{5}{4})=\log(\frac{15}{8})$)

71a $\log(x)=3\cdot\log(2)-2\cdot\log(3)$
$\log(x)=\log(2^3)-\log(3^2)$
$\log(x)=\log(8)-\log(9)$
$\log(x)=\log(\frac{8}{9})$
$x=\frac{8}{9}$.

71b $\log(x)=3+4\cdot\log(3)$
$\log(x)=\log(10^3)+\log(3^4)$
$\log(x)=\log(1000)+\log(81)$
$\log(x)=\log(81\,000)$
$x=81\,000$.

71c $3\cdot\log(x)+2=8$
$3\cdot\log(x)=6$
$\log(x)=2$
$x=10^2=100$.

71d $\log(x)=2-\log(2)$
$\log(x)=\log(10^2)-\log(2)$
$\log(x)=\log(100)-\log(2)$
$\log(x)=\log(\frac{100}{2})$
$x=50$.

71e $\log(x)=\log(x-1)+1$
$\log(x)=\log(x-1)+\log(10^1)$
$\log(x)=\log((x-1)\cdot 10)$
$x=10x-10$
$-9x=-10$
$x=\frac{10}{9}$.

71f $5\cdot\log(x+3)-2=13$
$5\cdot\log(x+3)=15$
$\log(x+3)=3$
$x+3=10^3=1000$
$x=997$.

72a $\log(x)=\log(6)-2\cdot\log(4)$
$\log(x)=\log(6)-\log(4^2)$
$\log(x)=\log(6)-\log(16)$
$\log(x)=\log(\frac{6}{16})$
$x=\frac{3}{8}$.

72b $\log(x)=1-3\cdot\log(2)$
$\log(x)=\log(10^1)-\log(2^3)$
$\log(x)=\log(10)-\log(8)$
$\log(x)=\log(\frac{10}{8})$
$x=\frac{5}{4}$.

72c $\log(x)=5-3\cdot\log(2)$
$\log(x)=\log(10^5)-\log(2^3)$
$\log(x)=\log(100\,000)-\log(8)$
$\log(x)=\log(\frac{100000}{8})$
$x=12\,500$.

72d $\log(x)=5-3\cdot\log(2)$
$\log(x)=\log(10^5)-\log(2^3)$
$\log(x)=\log(100\,000)-\log(8)$
$\log(x)=\log(\frac{100000}{8})$
$x=12\,500$.

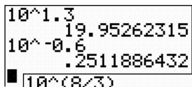
72e $\log(x)=5\cdot\log(2)-3\cdot\log(4)$
$\log(x)=\log(2^5)-\log(4^3)$
$\log(x)=\log(32)-\log(64)$
$\log(x)=\log(\frac{32}{64})$
$x=\frac{1}{2}$.

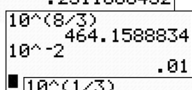
72f $\log(x+98)=\log(x-1)+2$
$\log(x+98)=\log(x-1)+\log(10^2)$
$\log(x)=\log((x-1)\cdot 100)$
$\log(x)=\log(100x-100)$
$x=100x-100$
$-99x=-100 \Rightarrow x=\frac{100}{99}$.

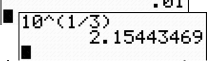
73a $\log(y)=p+5\Rightarrow y=10^{p+5}=10^p\cdot 10^5=100\,000\cdot 10^p$.

73b $\log(y)=q+1\Rightarrow y=10^{q+1}=10^q\cdot 10^1=10\cdot 10^q$.

73c $\log(y)=a-1\Rightarrow y=10^{a-1}=10^a\cdot 10^{-1}=\frac{1}{10}\cdot 10^a$.





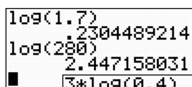


74a $\log(y)=1{,}3-0{,}6x\Rightarrow y=10^{1{,}3-0{,}6x}=10^{1{,}3}\cdot 10^{-0{,}6x}=10^{1{,}3}\cdot(10^{-0{,}6})^x\approx 20\cdot 0{,}25^x$.

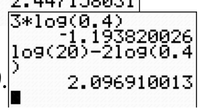
74b $3\cdot\log(P)=8-6t\Rightarrow\log(P)=\frac{8}{3}-2t\Rightarrow P=10^{\frac{8}{3}-2t}=10^{\frac{8}{3}}\cdot 10^{-2t}=10^{\frac{8}{3}}\cdot(10^{-2})^t\approx 460\cdot 0{,}01^t$.

74c $3\cdot\log(A)+12=t\Rightarrow 3\cdot\log(A)=t-12\Rightarrow\log(A)=\frac{1}{3}t-4\Rightarrow A=10^{\frac{1}{3}t-4}=10^{\frac{1}{3}t}\cdot 10^{-4}=\frac{1}{10^4}\cdot(10^{\frac{1}{3}})^t\approx 0{,}0001\cdot 2{,}15^t$.

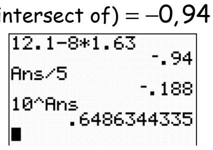
75a $N=280\cdot 1{,}7^t\Rightarrow\log(N)=\log(280\cdot 1{,}7^t)=\log(280)+\log(1{,}7^t)=\log(280)+t\cdot\log(1{,}7)\approx 0{,}23t+2{,}45$.



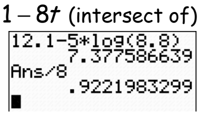
75b $N=20\cdot 0{,}4^{3t-2}\Rightarrow\log(N)=\log(20\cdot 0{,}4^{3t-2})=\log(20)+\log(0{,}4^{3t-2})$
$=\log(20)+(3t-2)\cdot\log(0{,}4)=\log(20)-2\cdot\log(0{,}4)+3t\cdot\log(0{,}4)\approx -1{,}19t+2{,}10$.



76a $t=1{,}63\Rightarrow 5\cdot\log(T)=12{,}1-8\cdot 1{,}63$ (intersect of) $=-0{,}94$
$\log(T)=-0{,}188$
$T=10^{-0{,}188}\approx 0{,}65$.



76b $T=8{,}8\Rightarrow 5\cdot\log(8{,}8)=12{,}1-8t$ (intersect of)
$8t=12{,}1-5\cdot\log(8{,}8)$ (: 8)
$t\approx 0{,}92$.



```
10^2.42
          263.0267992
10^-1.6
          .0251188643
```

76c $5\cdot\log(T)=12{,}1-8t \Rightarrow \log(T)=2{,}42-1{,}6t \Rightarrow T=10^{2{,}42-1{,}6t}=10^{2{,}42}\cdot 10^{-1{,}6t}=10^{2{,}42}\cdot(10^{-1{,}6})^t \approx 263\cdot 0{,}025^t$.

77a
$20\cdot\log(A)=5-100x$
$\log(A)=\frac{1}{4}-5x$
$A=10^{0{,}25-5x}$
$A=10^{0{,}25}\cdot 10^{-5x}$
$A=10^{0{,}25}\cdot(10^{-5})^x \approx 1{,}8\cdot 0{,}00001^x$.

```
10^0.25
          1.77827941
```

77b
$-5\cdot\log(y)=20-10x^2$
$\log(y)=-4+2x^2$
$y=10^{2x^2-4}$.

77c
$0{,}5\cdot\log(N)+3=5-2x$
$0{,}5\cdot\log(N)=2-2x$
$\log(N)=4-4x$
$N=10^{4-4x}$
$N=10^4\cdot 10^{-4x}$
$N=10^4\cdot(10^{-4})^x \approx 10\,000\cdot 0{,}0001^x$.

78 $\log(W)=0{,}38+0{,}008h \Rightarrow W=10^{0{,}38+0{,}008h}=10^{0{,}38}\cdot 10^{0{,}008h}=10^{0{,}38}\cdot(10^{0{,}008})^x \approx 2{,}4\cdot 1{,}0186^x$.

```
10^0.38
          2.398832919
10^0.008
          1.018591388
```

79a De bruine walvis is $\frac{100\,000}{10}=10\,000$ keer zo zwaar als de wasbeer.
De bruine walvis is $\frac{100\,000}{0{,}002}=50\,000\,000$ keer zo zwaar als de kolibrie.

```
100000/10
          10000
100000/0.002
          50000000
```

79b De getallenlijn zou $\frac{100\,000\,000}{1}=100\,000\,000$ mm $=100\,000$ m $=100$ km lang moeten worden.

79c De getallenlijn zou $\frac{100\,000}{1\,000}=100$ mm $=10$ cm lang moeten worden.
Bezwaar: de eerste 8 gewichten komen binnen de eerste 0,6 mm (praktisch niet uitvoerbaar).

80a A: 1,3; B: 7,5; C: 23; D: 55; E: 150 en F: 2400.

80b Op de verticale as staan lijntjes bij: 550; 210; 9,5 en 2,4; geen lijntjes bij: 310; 49; 1,25 en 0.

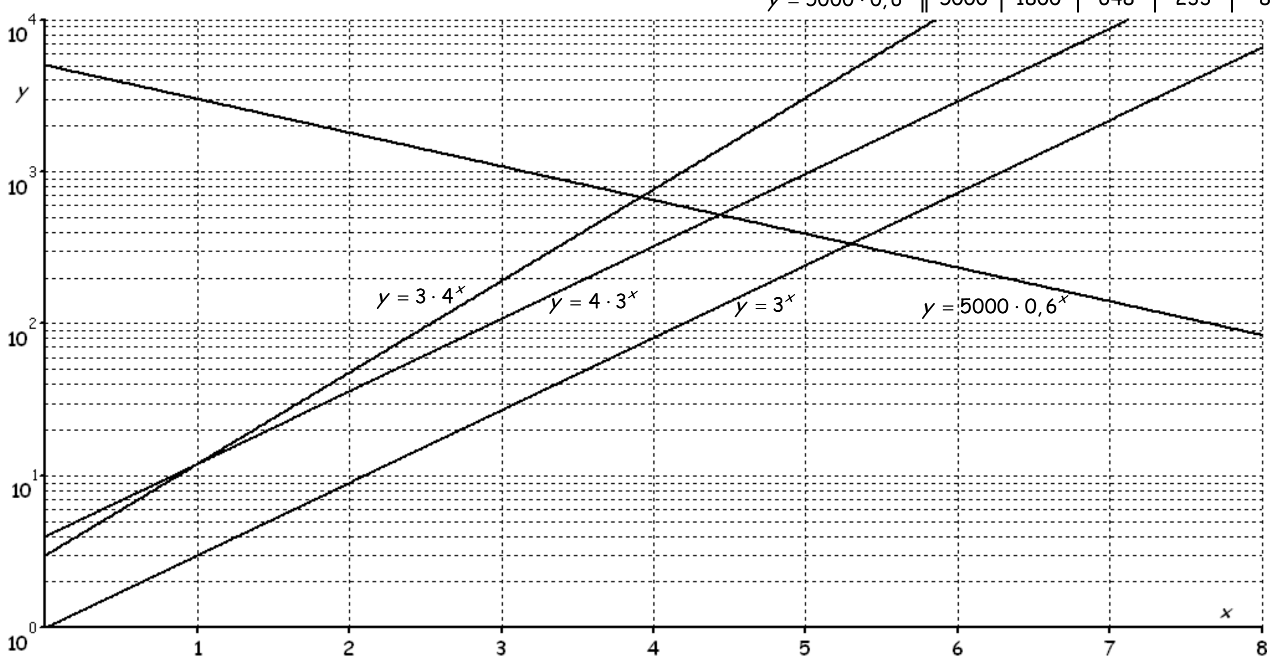
80c A: 1300; B: 7500; C: 23000; D: 55000; E: 150000 en F: 2400000. (1000 keer zo groot als bij 80a)

81a Zie de tabel hiernaast. (maak zelf eerst een tabel op de GR)

81b Zie de grafiek onder opgave 81. (de grafiek wordt een rechte lijn)
(gebruik het werkboek of vraag logaritmisch papier aan je docent)

81c Zie de tabel hiernaast en de grafiek hieronder.

x	0	2	4	6	8
$y=3^x$	1	9	81	729	6561
$y=4\cdot 3^x$	4	36	243	2916	26244
$y=3\cdot 4^x$	3	48	768	12288	196608
$y=5000\cdot 0{,}6^x$	5000	1800	648	233	84



82a Rechte lijn op enkelvoudig logaritmisch papier $\Rightarrow$ exponentiële functie $N=b\cdot g^t$.

$\left.\begin{matrix} t=1 \text{ en } N=3 \\ t=7 \text{ en } N=40 \end{matrix}\right\} \Rightarrow g_{6\text{ dagen}}=g^6=\frac{40}{3} \Rightarrow g_{\text{dag}}=g=\left(\frac{40}{3}\right)^{\frac{1}{6}}=1{,}53...$

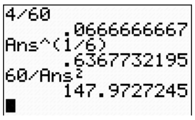
$\left.\begin{matrix} N=b\cdot 1{,}53...^t \\ t=1 \text{ en } N=3 \end{matrix}\right\} \Rightarrow b\cdot 1{,}53...^1=3 \Rightarrow b=\frac{3}{1{,}53...}\approx 1{,}95.$ Dus $N\approx 1{,}95\cdot 1{,}540^t$.

```
40/3
          13.33333333
Ans^(1/6)
          1.539890322
3/Ans
          1.948190698
```

82b Rechte lijn op enkelvoudig logaritmisch papier $\Rightarrow$ exponentiële functie $N = b \cdot g^t$.

$\left.\begin{array}{l} t = 2 \text{ en } N = 60 \\ t = 8 \text{ en } N = 4 \end{array}\right\} \Rightarrow g_{6 \text{ dagen}} = g^6 = \frac{4}{60} \Rightarrow g_{\text{dag}} = g = \left(\frac{4}{60}\right)^{\frac{1}{6}} = 0{,}636...$

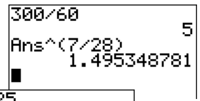
$\left.\begin{array}{l} N = b \cdot 0{,}636...^t \\ t = 2 \text{ en } N = 60 \end{array}\right\} \Rightarrow b \cdot 0{,}636...^2 = 60 \Rightarrow b = \frac{100}{0{,}636...^2} \approx 148.$ Dus $N \approx 148 \cdot 0{,}637^t$.



83a De grafieken van B en C zijn rechte lijnen $\Rightarrow$ bij de planten B en C is sprake van exponentiële groei.

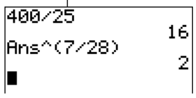
83bc Plant B: $L = b \cdot g^t$.

$\left.\begin{array}{l} t = 0 \text{ en } L = 60 \Rightarrow b = 60 \\ t = 28 \text{ en } L = 300 \end{array}\right\} \Rightarrow g_{28 \text{ dagen}} = \frac{300}{60} = 5 \Rightarrow g_{\text{week}} = g = 5^{\frac{7}{28}} \approx 1{,}50.$ Dus $L \approx 60 \cdot 1{,}50^t$.



Plant C: $L = b \cdot g^t$.

$\left.\begin{array}{l} t = 0 \text{ en } L = 25 \Rightarrow b = 25 \\ t = 28 \text{ en } L = 400 \end{array}\right\} \Rightarrow g_{28 \text{ dagen}} = \frac{400}{25} = 16 \Rightarrow g_{\text{week}} = g = 16^{\frac{7}{28}} = 2.$ Dus $L = 25 \cdot 2^t$.

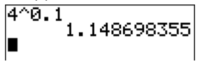


83d Plant E groeit exponentiëel $\Rightarrow$ een rechte lijn door (5,30) en (25,400). (doe dit zelf in het **werkboek**)

83e Plant F groeit exponentiëel $\Rightarrow$ rechte lijn door (10,50) evenwijdig met de grafiek van B. (zelf doen in het **werkboek**)

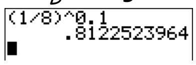
84a De grafiek van plant A is een rechte lijn op enkelvoudig logaritmisch papier $\Rightarrow$ exponentiële functie $N_A = b \cdot g^t$.

$\left.\begin{array}{l} t = 0 \text{ en } N_A = 5\,000 \Rightarrow b = 5\,000 \\ t = 10 \text{ en } N_A = 20\,000 \end{array}\right\} \Rightarrow g_{10 \text{ jaar}} = \frac{20000}{5000} = 4 \Rightarrow g = 4^{\frac{1}{10}} \approx 1{,}149.$ Dus $N_A \approx 5\,000 \cdot 1{,}149^t$.



De grafiek van plant B is een rechte lijn op enkelvoudig logaritmisch papier $\Rightarrow$ exponentiële functie $N_B = b \cdot g^t$.

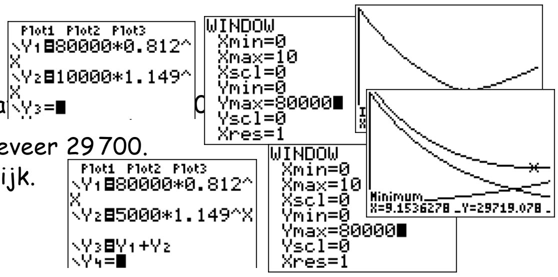
$\left.\begin{array}{l} t = 0 \text{ en } N_B = 80\,000 \Rightarrow b = 80\,000 \\ t = 10 \text{ en } N_B = 10\,000 \end{array}\right\} \Rightarrow g_{10 \text{ jaar}} = \frac{1}{8} \Rightarrow g = (\frac{1}{8})^{\frac{1}{10}} \approx 0{,}812.$ Dus $N_B \approx 80\,000 \cdot 0{,}812^t$.



84b $N_B = 2 \cdot N_A \Rightarrow 80\,000 \cdot 0{,}812^t = 10\,000 \cdot 1{,}149^t$ (intersect) $\Rightarrow t \approx 6{,}0.$

84c $N_C = 0{,}3 \cdot N_B = 0{,}3 \cdot 80\,000 \cdot 0{,}812^t = 24\,000 \cdot 0{,}812^t$.
De grafiek van N_C is dus evenwijdig met de grafiek van N_B en gaa

84d $N_A + N_B$ (minimum) heeft voor $t \approx 9{,}15$ het minimale aantal van ongeveer 29 700.
Het snijpunt van N_A en N_B ligt bij $t \approx 8 \Rightarrow$ Wesley heeft geen gelijk.

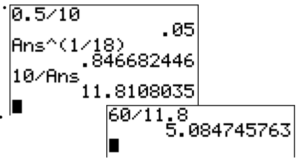


85a Teken met de gegevens uit de tabel 7 punten in het ▸werkboek.

85b Rechte lijn op enkelvoudig logaritmisch papier $\Rightarrow$ exponentiële functie $C = b \cdot g^t$.

$\left.\begin{array}{l} t = 1 \text{ en } C = 10 \\ t = 19 \text{ en } C = 0{,}5 \end{array}\right\} \Rightarrow g_{18 \text{ uur}} = g^{18} = \frac{0{,}5}{10} = 0{,}05 \Rightarrow g_{\text{uur}} = g = 0{,}05^{\frac{1}{18}} = 0{,}846...$

$\left.\begin{array}{l} c = b \cdot 0{,}846...^t \\ t = 1 \text{ en } C = 10 \end{array}\right\} \Rightarrow b \cdot 0{,}846...^1 = 10 \Rightarrow b = \frac{10}{0{,}846...} \approx 11{,}81.$ Dus $C \approx 11{,}81 \cdot 0{,}847^t$.



85c Bij x liter bloed is de concentratie op $t = 0$ gelijk aan $\frac{60}{x}$ mg/l $\Rightarrow \frac{60}{x} = \frac{11{,}8}{1} \Rightarrow 11{,}8x = 60 \Rightarrow x = \frac{60}{11{,}8} \approx 5{,}1$ (liter bloed).

86a Vul de tabel zelf in (gebruik TABLE).

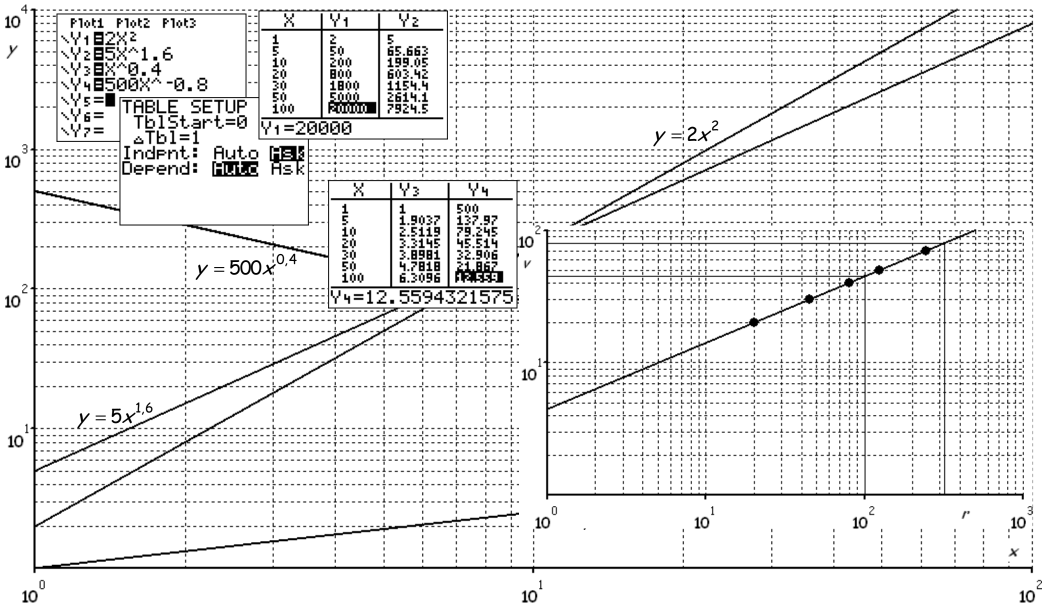
86bc Zie de grafieken hiernaast.

86d De grafieken (van machtsfuncties op dubbellogaritmisch papier) zijn rechte lijnen.

87ab Zie de grafiek (als inzet) hiernaast.

87b v is een machtsfunctie van $r \Rightarrow v = a \cdot r^p$.

87c $r = 100 \Rightarrow v \approx 44$ en $v = 80 \Rightarrow r = 320$.
(zo nauwkeurig niet af te lezen)



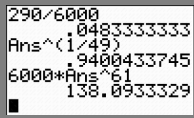
Bij het uitwerken van opgave 88 en 89 lees je op bladzijde 14 en 15 van het WERKBOEK-I hoe je Excel logaritmische schaalverdelingen kunt laten tekenen.

88a Zie de puntengrafiek onder deze opgave.

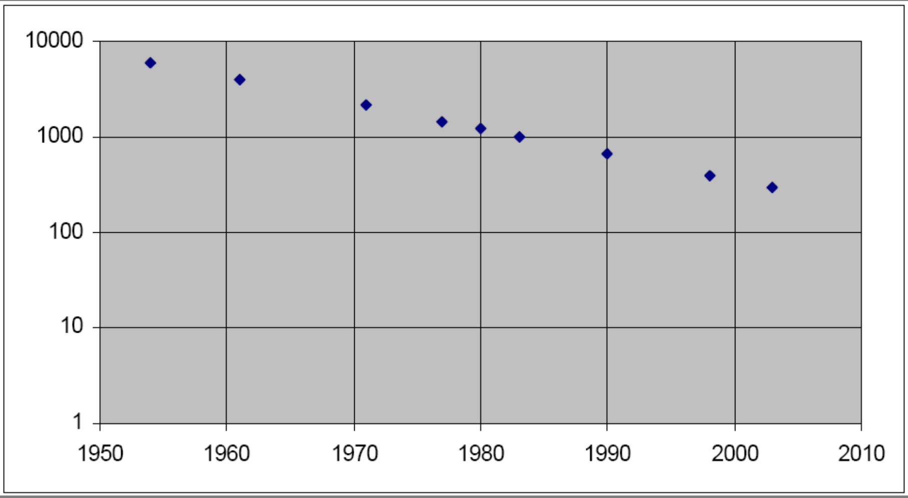
88b De punten liggen vrijwel op een rechte lijn $\Rightarrow$ exponentiële afname.

De groeifactor in 49 jaar is $\frac{290}{6000} \Rightarrow g_{\text{jaar}} = (\frac{290}{6000})^{\frac{1}{49}} \approx 0{,}94$.

De beginhoeveelheid is $6000 \Rightarrow N \approx 6000 \cdot 0{,}94^t$.



88c In 2015 is $t = 61 \Rightarrow N(61) \approx 6000 \cdot 0{,}94^{61} \approx 138$. Dus 138 broedparen.



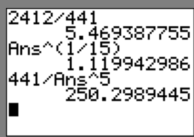
89a Zie de puntengrafiek onder deze opgave.

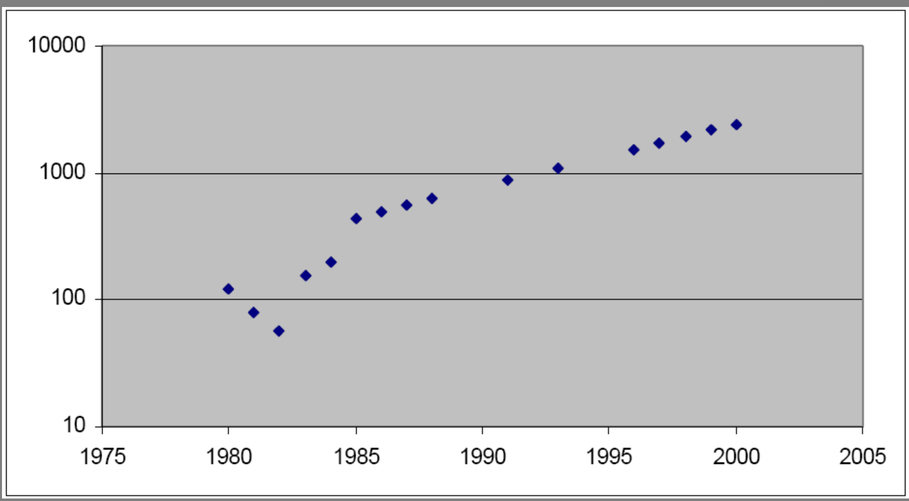
89b Vanaf 1990 liggen de punten vrijwel op een rechte lijn $\Rightarrow$ exponentiële groei.

89c Neem $t = 5$ voor 1985 $\Rightarrow$ punt (5, 441) en (20, 2 412).

De groeifactor in 15 jaar is $\frac{2412}{441} \Rightarrow g_{\text{jaar}} = (\frac{2412}{441})^{\frac{1}{15}} = 1{,}11...$

$\left.\begin{array}{l} N = b \cdot 1{,}11...^t \\ \text{door } (5, 441) \end{array}\right\} \Rightarrow 441 = b \cdot 1{,}11...^5 \Rightarrow b = \frac{441}{1{,}11...^5} \approx 250$. Dus $N \approx 250 \cdot 1{,}12^t$.





Diagnostische toets

D1 $K = aq + b$ met $a = \frac{\Delta K}{\Delta q} = \frac{1030-575}{460-200} = \frac{455}{260} = 1{,}75.$

$\left.\begin{array}{l} K = 1{,}75q + b \\ q = 200 \Rightarrow K = 575 \end{array}\right\} \Rightarrow 575 = 1{,}75 \cdot 200 + b \Rightarrow 575 - 1{,}75 \cdot 200 = b = 225.$ Dus $K = 1{,}75q + 225.$

```
455/260
                1.75
575-1.75*200
                 225
```

D2a $2{,}12t + 253 = 1{,}86t + 274$

$0{,}26t = 21$

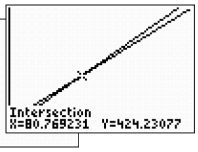
$t \approx 80{,}8.$

$K_{\text{I}} > K_{\text{II}}$ (zie een plot) $\Rightarrow t \geq 81$ (uur).

```
21/0.26
         80.76923077
```

```
Plot1 Plot2 Plot3
\Y1=2.12X+253
\Y2=1.86X+274
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=200
 Xscl=0
 Ymin=250
 Ymax=650
 Yscl=0
 Xres=1
```



D2b $K_{\text{I}} - K_{\text{II}} = 10$ (intersect of)

$2{,}12t + 253 - (1{,}86t + 274) = 10$

$0{,}26t - 21 = 10$

$0{,}26t = 31$

$t \approx 119$

$K_{\text{II}} - K_{\text{I}} = 10$ of $K_{\text{I}} - K_{\text{II}} = -10$ (intersect of)

$2{,}12t + 253 - (1{,}86t + 274) = -10$

$0{,}26t - 21 = -10$

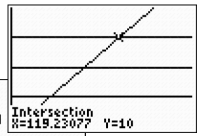
$0{,}26t = 11$

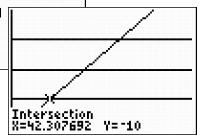
$t \approx 42.$

$-10 < K_{\text{I}} - K_{\text{II}} < 10$ (zie een plot) $\Rightarrow 42 < t < 119$ (uur).

```
Plot1 Plot2 Plot3
\Y1=2.12X+253
\Y2=1.86X+274
\Y3=Y1-Y2
\Y4=10
\Y5=-10
\Y6=
\Y7=
```

```
WINDOW
 Xmin=0
 Xmax=200
 Xscl=0
 Ymin=-20
 Ymax=20
 Yscl=0
 Xres=1
```





D3a $a + 2b = 36 \Rightarrow 2b = 36 - a \Rightarrow b = 18 - 0{,}5a.$ Dus $P = 20 - 0{,}3a + 0{,}8(18 - 0{,}5a) = 20 - 0{,}3a + 14{,}4 - 0{,}4a = 34{,}4 - 0{,}7a.$

D3b $b = 3a - 4 \Rightarrow P = 7a(3a-4) - 2a = 21a^2 - 28a - 2a = 21a^2 - 30a.$

D4a $\frac{1}{x^2 \cdot \sqrt{x}} = \frac{1}{x^2 \cdot x^{\frac{1}{2}}} = \frac{1}{x^{2\frac{1}{2}}} = x^{-2\frac{1}{2}}.$

D4b $(x^{2,4})^5 \cdot x^{3,8} = x^{12} \cdot x^{3,8} = x^{15,8}.$

D4c $(x^{2a-1})^4 \cdot x^3 = x^{8a-4} \cdot x^3 = x^{8a-1}.$

D5a $y = 2x^3 \cdot \sqrt{x} = 2x^3 \cdot x^{\frac{1}{2}} = x^{3\frac{1}{2}}.$

D5b $y = \frac{2}{x^3} \cdot \sqrt{x} = 2x^{-3} \cdot x^{\frac{1}{2}} = x^{-2\frac{1}{2}}.$

D5c $y = (3x^{-1,2})^3 \cdot 2x^{4,6} = 27 \cdot x^{-3,6} \cdot 2x^{4,6} = 54x.$

D6a $y = 5 \cdot x^{3,2} \Rightarrow x^{3,2} = \frac{1}{5}y \Rightarrow x = (\frac{1}{5}y)^{\frac{1}{3,2}} = (\frac{1}{5})^{\frac{1}{3,2}} \cdot y^{\frac{1}{3,2}} \approx 0{,}60 \cdot y^{0,31}.$

```
(1/5)^(1/3.
         .5848035476
1/3.2
               .3125
```

D6b $y = 5 \cdot \sqrt[3]{x^2} \Rightarrow x^{\frac{2}{3}} = \frac{1}{5}y \Rightarrow x = \left(\frac{1}{5}y\right)^{\frac{1}{\left(\frac{2}{3}\right)}} = \left(\frac{1}{5}\right)^{\frac{3}{2}} \cdot y^{\frac{3}{2}} \approx 0{,}09 \cdot y^{1,5}.$

```
1/(2/3)
                 1.5
(1/5)^(3/2)
         .0894427191
```

D6c $y = 5 \cdot (3x)^{2,1} \Rightarrow (3x)^{2,1} = \frac{1}{5}y \Rightarrow 3x = (\frac{1}{5}y)^{\frac{1}{2,1}} \Rightarrow x = \frac{1}{3} \cdot (\frac{1}{5})^{\frac{1}{2,1}} \cdot y^{\frac{1}{2,1}} \approx 0{,}15 \cdot y^{0,48}.$

```
1/3*(1/5)^(1/2.1
)
         .1548944604
1/2.1
         .4761904762
```

D7a $N = 15 \cdot 2{,}6^{3t-1} = 15 \cdot 2{,}6^{3t} \cdot 2{,}6^{-1} = 15 \cdot 2{,}6^{-1} \cdot (2{,}6^3)^t \approx 5{,}77 \cdot 17{,}58^t.$

```
15*2.6^-1
         5.769230769
2.6^3
              17.576
```

D7b $N = 600 \cdot 0{,}8^{4t-3} = 600 \cdot 0{,}8^{4t} \cdot 0{,}8^{-3} = 600 \cdot 0{,}8^{-3} \cdot (0{,}8^4)^t \approx 1172 \cdot 0{,}41^t.$

```
600*0.8^-3
            1171.875
0.8^4
               .4096
```

D7c $N = 500 \cdot 0{,}2^{0,05t-2} = 500 \cdot 0{,}2^{0,05t} \cdot 0{,}2^{-2} = 500 \cdot 0{,}2^{-2} \cdot (0{,}2^{0,05})^t \approx 12\,500 \cdot 0{,}92^t.$

```
500*0.2^-2
               12500
0.2^0.05
         .9226808346
```

D8a $g_{\text{dag}} = 1{,}36 \Rightarrow g_{\text{week}} = 1{,}36^7 \approx 8{,}61 = 861\%.$ Het groeipercentage per week is 761%.

```
1.36^7
         8.605425688
Ans*100-100
         760.5425688
```

D8b $g_{\text{dag}} = 1{,}36 \Rightarrow g_{\text{uur}} = 1{,}36^{\frac{1}{24}} \approx 1{,}013 = 101{,}3\%.$ Het groeipercentage per uur is 1,3%.

```
1.36^(1/24)
         1.012894286
Ans*100-100
         1.289428602
```

D9a $g_{10\,\text{jaar}} = 0{,}75 \Rightarrow g_{\text{jaar}} = 0{,}75^{\frac{1}{10}} \approx 0{,}972 = 97{,}2\%.$ De afname per jaar is 2,8%.

```
0.75^0.1
         .9716416579
Ans*100-100
        -2.835834214
```

D9b $g_{10\,\text{jaar}} = 0{,}75 \Rightarrow g_{25\,\text{jaar}} = 0{,}75^{2,5} \approx 0{,}487 = 48{,}7\%.$ De afname per jaar is 51,3%.

```
0.75^2.5
         .4871392896
Ans*100-100
        -51.28607104
```

D10a $N_1 = at + b$ met $a = \frac{\Delta N}{\Delta t} = \frac{3900-3000}{8-5} = \frac{900}{3} = 300.$

$\left.\begin{array}{l} N_1 = 300t + b \\ t = 5 \Rightarrow N_1 = 3000 \end{array}\right\} \Rightarrow 3000 = 300 \cdot 5 + b \Rightarrow b = 3000 - 1500 = 1500.$ Dus $N_1 = 300t + 1500.$

D10b $N_2 = b \cdot g^t$ met $g^{8-5} = g^3 = \frac{3900}{3000} = 1{,}3 \Rightarrow g = 1{,}3^{\frac{1}{3}} = 1{,}091...$

$\left.\begin{array}{l} N_2 = b \cdot 1{,}091...^t \\ t = 5 \Rightarrow N_2 = 3000 \end{array}\right\} \Rightarrow 3000 = b \cdot 1{,}091...^5 \Rightarrow b = \frac{3000}{1{,}091...^5} \approx 95.$ Dus $N_2 \approx 1940 \cdot 1{,}0914^t.$

```
3900/3000
                 1.3
Ans^(1/3)
         1.091392883
3000/Ans^5
         1937.383816
```

D10c $N_1 = 300t + 1500 = 9000 \Rightarrow 300t = 7500 \Rightarrow t = 25$ en $N_2(25) \approx 1940 \cdot 1{,}0914^{25} \approx 17\,270.$

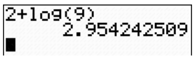
```
7500/300
                  25
1940*1.0914^25
         17274.30034
```

D11a $3+2\cdot\log(x)=7$
$2\cdot\log(x)=4$
$\log(x)=2 \Rightarrow x=10^2=100.$

D11b $\log(2x-300)=4$
$2x-300=10^4=10\,000$
$2x=10\,300 \Rightarrow x=5150.$

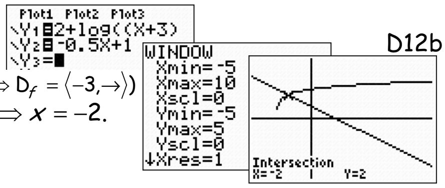
D11c $5+3\cdot\log(x)=2$
$3\cdot\log(x)=-3$
$\log(x)=-1 \Rightarrow x=10^{-1}=\frac{1}{10}.$

D12a $f(6)=2+\log(6+3)\approx 2{,}95.$



D12b $f(x)=5$
$2+\log(x+3)=5$
$\log(x+3)=3$
$x+3=10^3=1000$
$x=997.$

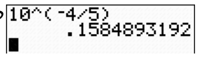
D12c $f(x)=2+\log(x+3)$ (voorwaarde: $x+3>0 \Rightarrow x>-3 \Rightarrow D_f=\langle -3,\rightarrow\rangle$)
$f(x)=g(x) \Rightarrow 2+\log(x+3)=-0{,}5x+1$ (intersect) $\Rightarrow x=-2.$
$f(x)<g(x)$ (zie een plot en D_f) $\Rightarrow -3<x<-2.$



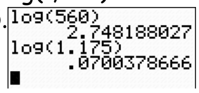
D13a $5\cdot {}^3\log(4)-\frac{1}{2}\cdot {}^3\log(16)={}^3\log(4^5)-{}^3\log(16^{\frac{1}{2}})={}^3\log(1024)-{}^3\log(\sqrt{16})={}^3\log(1024)-{}^3\log(4)={}^3\log(\frac{1024}{4})={}^3\log(256).$

D13b $\log(x+145)=1+\log(x+10)$
$\log(x+145)=\log(10^1)+\log(x+10)$
$\log(x+145)=\log(10\cdot(x+10))$
$x+145=10\cdot(x+10)$
$x+145=10x+100$
$-9x=-45$
$x=5.$

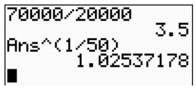
D13c $5\cdot\log(N)=10-4P$
$\log(N)=2-\frac{4}{5}P$
$N=10^{2-\frac{4}{5}P}=10^2\cdot 10^{-\frac{4}{5}P}$
$N=100\cdot(10^{-\frac{4}{5}})^P$
$N\approx 100\cdot 0{,}158^P.$



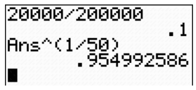
D13d $F=560\cdot 1{,}175^t$
$\log(F)=\log(560\cdot 1{,}175^t)$
$\log(F)=\log(560)+\log(1{,}175^t)$
$\log(F)=\log(560)+t\cdot\log(1{,}175)$
$\log(F)\approx 0{,}07t+2{,}75.$



D14a Rechte lijn op (met langs Eén as een logaritmische schaal $\Rightarrow$ enkelvoudig) logaritmisch papier $\Rightarrow N=b\cdot g^t$.
De grafiek van buurgemeente A is
een rechte lijn door (0, 20 000) en (50, 70 000) $\Rightarrow g^{50}=\frac{70000}{20000}=3{,}5 \Rightarrow g=3{,}5^{\frac{1}{50}}\approx 1{,}025.$

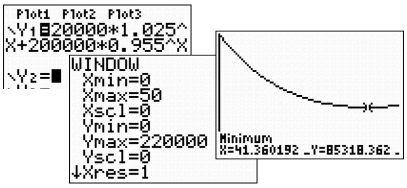


De formule is $N_A=20\,000\cdot 1{,}025^t$.
De grafiek van buurgemeente B is
een rechte lijn door (0, 200 000) en (50, 20 000) $\Rightarrow g^{50}=\frac{20000}{200000}=0{,}1 \Rightarrow g=0{,}1^{\frac{1}{50}}\approx 0{,}955.$



De formule is $N_B=200\,000\cdot 0{,}955^t$.

D14b $S=20\,000\cdot 1{,}025^t+200\,000\cdot 0{,}955^t.$
De optie minimum geeft $t\approx 41$,... en $S\approx 85\,318$.
Dus in $1950+41=1991$ is het aantan minimaal.
Er zijn dan in A en B samen 85 318 inwoners.



Gemengde opgaven 13. Algebraïsche vaardigheden

G1a $g_{25\text{ jaar}} = g^{25} = \frac{6,8}{10,9} \Rightarrow g = \left(\frac{6,8}{10,9}\right)^{\frac{1}{25}} = 0,981...$

$\left.\begin{array}{l} N = b \cdot 0,981...^t \\ t = 5 \text{ en } N = 10,9 \end{array}\right\} \Rightarrow b \cdot 0,981...^5 = 10,9 \Rightarrow b = \frac{10,9}{0,981...^5} \approx 11,98.$ Dus $N \approx 12,0 \cdot 0,981^t$.

```
(6.8/10.9)^(1/25
)
          .9813033842
10.9/Ans^5
          11.97870901
```

G1b Bij 2015 hoort $t = 45 \Rightarrow N = 12,0 \cdot 0,981^{45} \approx 5,1$ (miljoen broedparen).

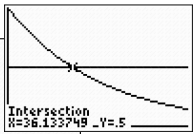
G1c Bij 1960 hoort $t = -10 \Rightarrow N = 12,0 \cdot 0,981^{-10} \approx 14,5$ (miljoen broedparen).

```
12*0.981^45
          5.06158551
12*0.981^-10
          14.53754835
```

G1d $0,981^t = \frac{1}{2}$ (intersect) $\Rightarrow t \approx 36,1$.
Halveringstijd is 36,1 jaar.

```
Plot1 Plot2 Plot3
\Y1=0.981^X
\Y2=1/2
\Y3=
```

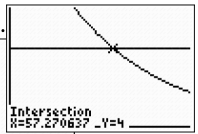
```
WINDOW
 Xmin=0
 Xmax=100
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
↓Xres=1
```



G1e $12,0 \cdot 0,981^t = 4$ (intersect) $\Rightarrow t \approx 57$.
Dus in $1970 + 57 = 2027$.

```
Plot1 Plot2 Plot3
\Y1=12*0.981^X
\Y2=4
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=100
 Xscl=0
 Ymin=0
 Ymax=6
 Yscl=0
↓Xres=1
```



G2a $g_{4\text{ jaar}} = g^4 = \frac{759}{117} \Rightarrow g = \left(\frac{759}{117}\right)^{\frac{1}{4}} = 1,59...$

```
(759/117)^(1/4)
          1.595930514
117/Ans^4
          18.03557312
```

$\left.\begin{array}{l} N = b \cdot 1,59...^t \\ t = 4 \text{ en } N = 117 \end{array}\right\} \Rightarrow b \cdot 1,59...^4 = 117 \Rightarrow b = \frac{117}{1,59...^4} \approx 18.$

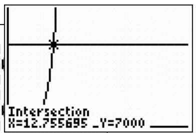
Dus $N \approx 18 \cdot 1,596^t$.

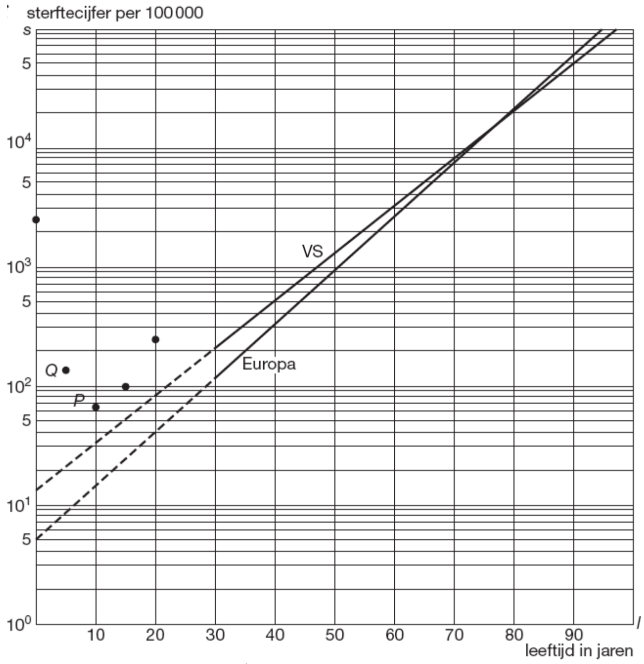
G2b $g \approx 1,596 = 159,6\% \Rightarrow$ de toename per jaar is 59,6%.

G2c $18 \cdot 1,596^t = 7\,000$ (intersect) $\Rightarrow t = 12,...$
Dus in $1995 + 12 = 2007$.

```
Plot1 Plot2 Plot3
\Y1=18*1.596^X
\Y2=7000
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=5
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
↓Xres=1
```





G3a Lees in de grafiek hiernaast af: $s \approx 65$.

G3b Bij $l = 0$ (leeftijd 0 jaar $\Rightarrow$ van 0 tot 1 jaar)
hoort $s \approx 2\,500$ (zie het dikke punt op de verticale as).
Het gevraagde percentage is $\frac{2500}{100000} \times 100\% = 2,5\%$.

```
2500/100000*100
                2.5
```

G3c Zie het punt $Q(5, 126)$ in de grafiek hiernaast.

G3d Rechte lijn op logaritmisch papier $\Rightarrow s = b \cdot g^l$.

De grafiek gaat door (30, 200) en (90, 50 000) $\Rightarrow g^{60} = \frac{50000}{200} = 250 \Rightarrow g = 250^{\frac{1}{60}} = 1,096...$

$\left.\begin{array}{l} s_{VS} = b \cdot 1,096...^l \\ l = 30 \text{ en } N = 200 \end{array}\right\} \Rightarrow b \cdot 1,096...^{30} = 200 \Rightarrow b = \frac{200}{1,096...^{30}} \approx 12,65.$ Dus $s_{VS} \approx 12,65 \cdot 1,096^l$.

```
250^(1/60)
          1.096391517
200/Ans^30
          12.64911064
```

G3e $\log(s_{EU}) = 0,045l + 0,68 \Rightarrow s_{EU} = 10^{0,045l+0,68} = 10^{0,045l} \cdot 10^{0,68} = 10^{0,68} \cdot (10^{0,045})^l \approx 4,8 \cdot 1,11^l$.

```
10^0.68
          4.786300923
10^0.045
          1.109174815
```

G3f $l = 30 \Rightarrow s_{EU} \approx 4,8 \cdot 1,11^{30} \approx 110$ en $l = 90 \Rightarrow s_{EU} \approx 4,8 \cdot 1,11^{90}$.
Teken de lijn door (30, 110) en (90, 57 600) (zie in de figuur hierboven).

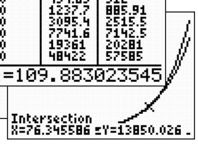
G3g $s_{VS} = s_{EU}$ (intersect/aflezen in figuur) $\Rightarrow l \approx 76$ (jaar).

```
Plot1 Plot2 Plot3
\Y1=12.65*1.096^
X
\Y2=4.8*1.11^X
\Y3=
```

```
WINDOW
 Xmin=30
 Xmax=90
 Xscl=0
 Ymin=0
 Ymax=60000
 Yscl=0
↓Xres=1
```

X	Y1	Y2
30	197.88	109.88
40	494.89	312
50	1237.7	885.91
60	3095.4	2515.5
70	7741.6	7142.5
80	19361	20281
90	48422	57585

Y2=109.883023545



G4a $L = 75 \Rightarrow 20 \cdot \log(N) = 202 - \frac{4}{3} \cdot 75 = 102 \Rightarrow \log(N) = 5,1 \Rightarrow N = 10^{5,1} \approx 125\,893$.

$L = 70 \Rightarrow 20 \cdot \log(N) = 202 - \frac{4}{3} \cdot 70 \Rightarrow \log(N) = 5,43... \Rightarrow N = 10^{5,43...} \approx 271\,227$.

271 227 is meer dan het dubbele van 125 893.

```
202-4/3*75
                102
Ans/20
                5.1
10^5.1
          125892.5412
```

```
202-4/3*70
          108.6666667
Ans/20
          5.433333333
10^Ans
          271227.2579
```

G4b $20 \cdot \log(500\,000) = 202 - \frac{4}{3}L$

$113,... = 202 - \frac{4}{3}L$

$\frac{4}{3}L = 88,...$

$L \approx 66$.

```
20*log(500000)
          113.9794001
202-Ans
          88.02059991
Ans/(4/3)
          66.01544993
```

G4c $202 - \frac{4}{3}L = 248 - 2L$

$\frac{2}{3}L = 46$

$L = 69$.

```
46/2*3
                69
```

G4d $20\log(N) = 248 - 2L$

$\log(N) = 12,4 - 0,1L$

$N = 10^{12,4-0,1L}$.

```
Plot1 Plot2 Plot3
\Y1=10^(12.4-0.1
X)
\Y2=Y1/1000
\Y3=
```

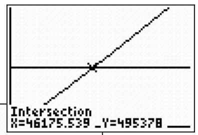
X	Y1	Y2
63	1.26E6	1258.9
64	1E6	1000
65	794328	794.33
66	630957	630.96
67	501187	501.19
68	398107	398.11
69	316228	316.23

X=63

G4e Vul formule (2) in op de GR en gebruik TABLE (de kolom onder Y2) voor een schets.
Bij afname van L (voor $L < 69$) geeft formule (2) een hogere N_{max} dan formule (1) $\Rightarrow$ vaker lawaai.

G5a De verdeling van ELK ligt ten opzichte van AZM naar links $\Rightarrow$ serie I hoort bij ELK.

G5b Rechts van de mediaan liggen de gegevens verder uit elkaar dan links van de mediaan.
Dus de mediaan is kleiner dan het gemiddelde. (de staart naar rechts trekt gemiddelde naar rechts)




```
Plot1 Plot2 Plot3
\Y1=2.3*X*log(X)
\Y2=495378
\Y3=

WINDOW
Xmin=0
Xmax=100000
Xscl=0
Ymin=0
Ymax=1000000
Yscl=0
Xres=1
```

G5c $2{,}3 \cdot C \cdot \log(C) = 495\,378$ (intersect) $\Rightarrow C \approx 46\,000$.

G5d Bij $r = 100$ is het verschil (ongeveer 1900 – 880 =) 1000.
Bij $r = 500$ is het verschil (ongeveer 350 – 180 =) 200.
Dus het verschil bij $r = 100$ is groter dan bij $r = 500$.

```
88000/100      880
88000/500      176
```

```
330/55*100     600
330/75*100     440
330/600*100     55
```

G6a $EEI = \dfrac{330}{standaardenergieverbruik} \cdot 100\% = 55\% \Rightarrow standaardenergieverbruik = 330 \cdot 100 : 55 = 660$ (kWh/jaar).

G6b De aanschafprijs van de Icebox (EEI = 60% $\Rightarrow$ klasse A) is met subsidie € 945.
Dus de Icebox is € 150 duurder.
De Icebox is in verbruik $(480 - 360) \cdot 0{,}2 =$ € 24 per jaar goedkoper.
Het duurt $\frac{150}{24} = 6{,}25$ jaar (dus 6 jaar en 3 maanden) $\Rightarrow$ de extra investering is in april 2006 terugverdiend.

```
995-50         945
945-795        150
480-360        120

120*0.2         24
150/24        6.25
Ans-6          .25
Ans*12
```

G6c $standaardenergieverbruik = m \cdot GV + n = m \cdot (K + s \cdot V) + n = mK + msV + n = a \cdot K + b \cdot V + c$.
Dus $a = m = 0{,}450$; $b = ms = 0{,}450 \cdot 1{,}85 = 0{,}8325$ en $c = n = 245$ (gebruik de onderste regel van de tabel).

```
0.450*1.85   .8325
```

G6d $standaardenergieverbruik = 0{,}657 \cdot K + 1{,}41255 \cdot V + 235$ met $K = 4 \times V \Rightarrow$
$standaardenergieverbruik = 0{,}657 \cdot 4 \cdot V + 1{,}41255 \cdot V + 235 = 4{,}04055 \cdot V + 235$.
klasse A $\Rightarrow EEI < 55\% \Rightarrow$ *standaardenergieverbruik* moet meer dan 340 zijn.
$4{,}04055 \cdot V + 235 > 340 \Rightarrow V$ heeft een inhoud van (tenminste) 26 liter.

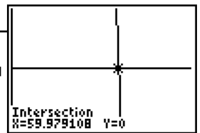
```
0.657*4          2.628
Ans+1.41255    4.04055
187*100/55         340

340-235            105
105/4.04055 25.98656124
```

G7a Bij de mannen een relatieve toename van 7% in 1950 naar 10% in 1960 $\Rightarrow$ een factor van $\frac{10}{7} = 1\frac{3}{7}$.
Bij de vrouwen een relatieve toename van 4% in 1950 naar 6% in 1960 $\Rightarrow$ een factor van 1,5.
Dus bij de mannen niet groter dan bi de vrouwen.
of bij de mannen 283 dodelijke slachtoffers in 1950 tegen 433 in 1960 $\Rightarrow$ een toename van 53%
bij de vrouwen 162 dodelijke slachtoffers in 1950 tegen 260 in 1960 $\Rightarrow$ een toename van 60%
de conclusie: nee

```
10/7   1.428571429
6/4            1.5

0.07*4046   283.22
0.10*4332    433.2
433/283 1.530035336

0.04*4046   161.84
0.06*4332   259.92
260/162 1.604938272
```

G7b $B = 5{,}4 \Rightarrow N_{max} = \dfrac{8289{,}3 \cdot (1{,}778 - \log(5{,}4))}{5{,}4} \approx 1605$.
$1740 > N_{max} = 1605$, dus de weg <u>voldoet niet</u> aan de veilige norm.

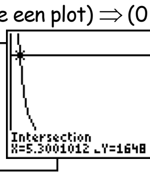
```
8289.3*(1.778-lo
g(5.4))/5.4
       1605.063668

Plot1 Plot2 Plot3
\Y1=(8289.3*(1.7
78-log(X))/X
\Y2=0
\Y3=
\Y4=

WINDOW
Xmin=0
Xmax=100
Xscl=0
Ymin=-1
Ymax=1
Yscl=0
Xres=1
```



G7c $N_{max} = 0$ (intersect/algebraïsch) $\Rightarrow B = 59{,}98$. Je vindt: $N_{max} > 0$ (zie een plot) $\Rightarrow (0 <) B < 59{,}98$.

G7d $N_{max} = 1648$ (intersect) $\Rightarrow B = 5{,}3$.
$N_{max}(5{,}3 - 0{,}5) \Rightarrow N_{max} = 1894$.
N_{max} neemt met 246 toe.

```
Plot1 Plot2 Plot3
\Y1=(8289.3*(1.7
78-log(X))/X
\Y2=1648
\Y3=

WINDOW
Xmin=0
Xmax=100
Xscl=0
Ymin=0
Ymax=2000
Yscl=0
Xres=1
```




```
X          5.300101165
Y1(X-0.5)
           1893.978111
Ans-1648
           245.9781114
```

G8a De gemiddelde jaarlijkse neerslag is in beide plaatsen gelijk.
De standaardafwijking (en dus de spreiding) in Winterswijk is groter.
Dus de kans op meer dan 950 mm neerslag in Winterswijk is groter dan in Hoofddorp.

```
normalcdf(950,10
^99,768,136)
       .0904099352
normalcdf(950,10
^99,768,127)
        .075918838
```

G8b normalcdf$(950, 10^{99}, 753, 106) \approx 0{,}032$.

```
normalcdf(950,10
^99,753,106)
       .0315496282
```

G8c Aflezen van twee punten op de trendlijn, bijvoorbeeld (0,720) en (100,800).
$N = at + b$ met $a = \frac{\Delta N}{\Delta t} = \frac{800-720}{100} = \frac{80}{100} = 0{,}8$. Dus $N = 0{,}8t + 720$.
$N = 0{,}8t + 720 = 850 \Rightarrow 0{,}8t = 130 \Rightarrow t = 162{,}5$.
Voor het eerst groter dan 850 mm in het jaar 2063.

```
850-720        130
Ans/0.8      162.5
Ans+1900    2062.5
```

```
47*46*45*44*43/9
4/93/92/91/90
       .0279451731
47 nCr 5/94 nCr
5
       .0279451731
```

G8d $P(X = 5) = \frac{47}{94} \cdot \frac{46}{93} \cdot \frac{45}{92} \cdot \frac{44}{91} \cdot \frac{43}{90}$ of $\dfrac{\binom{47}{5}}{\binom{94}{5}} \approx 0{,}0279$.

G8e Maak voor de Bilt in 2001
eerst de tabel hiernaast.

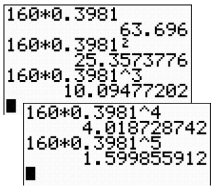
grenswaarde	> 30	> 40	> 50	> 60	> 70	> 80	> 90	> 100	> 110	> 120	> 130
aantal maanden	11	11	10	9	9	7	3	2	2	1	1

Vergelijk deze tabel met de gemiddeld aantal maanden per jaar.
Je ziet dat 2001 voor 10 grenswaarden een grotere waarde heeft $\Rightarrow$ 2001 was een extreem nat jaar.

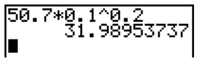
G9a Bijvoorbeeld $25{,}4 = 2 \cdot 12{,}7 \Rightarrow 2 \cdot 19{,}0 = 38{,}0$.
(of bijv. 0,5 keer het getal bij $KH = 12 \Rightarrow 38{,}1$)

```
25.4/2        12.7
2*19            38
1/2*76.1     38.05
```

G9b $g_{\text{per } pH=1} = 0{,}1 \Rightarrow g_{\text{per } pH=0{,}4} = 0{,}1^{0{,}4} \approx 0{,}3981.$
$pH = 6{,}4 \Rightarrow C = 160{,}0 \cdot 0{,}3981 \approx 63{,}7.$
$pH = 6{,}8 \Rightarrow C = 160{,}0 \cdot 0{,}3981^2 \approx 25{,}4.$
$pH = 7{,}2 \Rightarrow C = 160{,}0 \cdot 0{,}3981^3 \approx 10{,}1.$
$pH = 7{,}6 \Rightarrow C = 160{,}0 \cdot 0{,}3981^4 \approx 4{,}0.$
$pH = 8{,}0 \Rightarrow C = 160{,}0 \cdot 0{,}3981^5 \approx 1{,}6.$

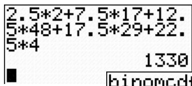


G9c $\left.\begin{array}{l} pH = 7 \Rightarrow \text{aan voorwaarde II is voldaan} \\ KH = 8 \Rightarrow \text{aan voorwaarde I is voldaan} \\ C = 50{,}7 \cdot 0{,}1^{0{,}2} \approx 32 \Rightarrow \text{aan voorwaarde III is voldaan} \end{array}\right\} \Rightarrow$ dit vijverwater is van goede kwaliteit.

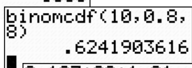


G9d Omdat $C \geq 10$ ligt het gebied links van de kromme.
Het gebied ligt tussen de horizontale grenslijnen $KH = 6$ en $KH = 10$.
Het gebied ligt rechts van de verticale grenslijn $pH = 7$. (kleur nu dit gebied aan in het werkboek)

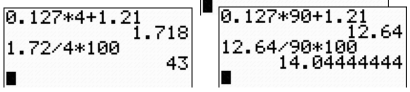
G10a De klassenmiddens zijn 2,5; 7,5; 12,5; 17,5 en 22,5.
De schatting is $2{,}5 \cdot 2 + 7{,}5 \cdot 17 + 12{,}5 \cdot 48 + 17{,}5 \cdot 29 + 22{,}5 \cdot 4 = 1330$ (euro).



G10b $X =$ het aantal klanten dat een fooi geeft. $P(X \leq 8) = \text{binomcdf}(10, 0.8, 8) \approx 0{,}624.$

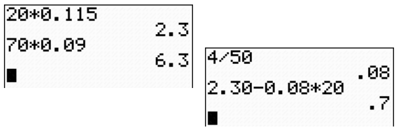


G10c $R = 4 \Rightarrow F = 0{,}172 \cdot 4 + 1{,}21 \approx 1{,}72$ en dat is $\frac{1{,}72}{4} \times 100 = 43\%.$
$R = 90 \Rightarrow F = 0{,}172 \cdot 90 + 1{,}21 = 12{,}64$ en dat is $\frac{12{,}64}{90} \times 100 \approx 14\%.$

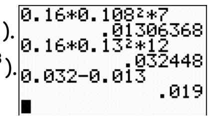


G10d Het vaste gedeelte van 1,21 heb je bij manier II vier keer en bij I maar één keer.
Het variabele gedeelte is bij beide manieren gelijk. Dus manier II levert het grootste bedrag op.

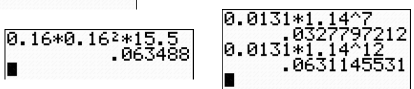
G10e $R = 20 \Rightarrow F = 0{,}115 \cdot 20 = 2{,}30$ en $R = 70 \Rightarrow F = 0{,}09 \cdot 70 = 6{,}30.$
$F = aR + b$ met $a = \frac{6{,}30 - 2{,}30}{70 - 20} = 0{,}08.$
$\left.\begin{array}{l} F = 0{,}08R + b \\ R = 20 \Rightarrow F = 2{,}30 \end{array}\right\} \Rightarrow 2{,}30 = 0{,}08 \cdot 20 + b \Rightarrow b = 0{,}7.$



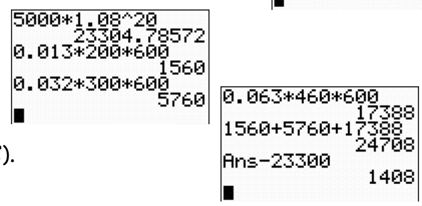
G11a Bij 8 jaar hoort $D = 0{,}108$ en $L = 7 \Rightarrow M = 0{,}16 \cdot 0{,}108^2 \cdot 7 \approx 0{,}013$ (m^3).
Bij 15 jaar hoort $D = 0{,}13$ en $L = 12 \Rightarrow M = 0{,}16 \cdot 0{,}13^2 \cdot 12 \approx 0{,}032$ (m^3).
Het verschil is $0{,}032 - 0{,}013 = 0{,}019$ (m^3).



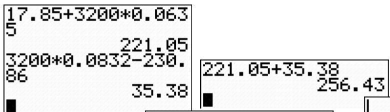
G11b Bij 20 jaar hoort $D = 0{,}16$ en $L = 15{,}5 \Rightarrow M = 0{,}16 \cdot 0{,}16^2 \cdot 15{,}5 \approx 0{,}063$ (m^3).
$M(8) \cdot 1{,}14^7 \approx 0{,}0328 \approx M(15)$ en $M(8) \cdot 1{,}14^{12} \approx 0{,}0631 \approx M(20).$



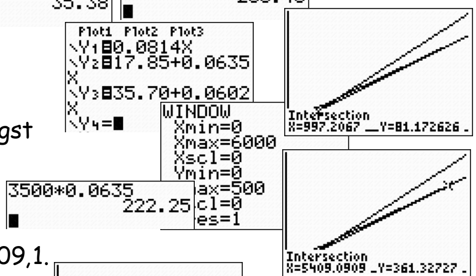
G11c De spaarrekening levert $5\,000 \cdot 1{,}08^{20} \approx 23\,300$ (€) op.
De houtopbrengst na 8 jaar is (ongeveer) $0{,}013 \cdot 200 \cdot 600 = 1560$ (€).
De houtopbrengst na 15 jaar is (ongeveer) $0{,}032 \cdot 300 \cdot 600 = 5\,760$ (€).
De houtopbrengst na 20 jaar is (ongeveer) $0{,}063 \cdot 460 \cdot 600 = 17\,388$ (€).
De totale opbrengst is naar verwachting tenminste (ongeveer) 24 700 (€).
Dat is (ongeveer) 1400 (€) meer (of 1600 zonder tussentijds afronden)



G12a De elektriciteitskosten zijn $17{,}85 + 3200 \cdot 0{,}0635 = 221{,}05$ (€).
De energiebelasting is $3200 \cdot 0{,}0832 - 230{,}86 = 35{,}38$ (€).
Het rekening is $221{,}05 + 35{,}38 = 256{,}43$ (€).



G12b $0{,}0814x = 17{,}85 + 0{,}0635x$ (algebraïsch of intersect) $\Rightarrow x \approx 997{,}2.$
$17{,}85 + 0{,}0635x = 35{,}70 + 0{,}0602x$ (algebraïsch of intersect) $\Rightarrow x \approx 5\,409{,}1.$
Bij een verbruik van 0 tot en met 997 kWh is Budget het voordeligst,
bij een verbruik van 998 tot en met 5409 kWh is Standaard het voordeligst
en bij een verbruik van minstens 5410 kWh is Plus het voordeligst.



G12c Bij enkeltarief zijn de kosten $3500 \cdot 0{,}0635 = 222{,}25$ (zonder vaste kosten).
Bij laag-/normaaltarief zijn de kosten $0{,}0419 \cdot x + 0{,}0749 \cdot (3500 - x).$
$0{,}0419 \cdot x + 0{,}0749 \cdot (3500 - x) = 222{,}25$ (algebraïsch of intersect) $\Rightarrow x = 1\,209{,}1.$
Er moet tenmiste 1210 (of 1 209) kWh volgens laagtarief worden verbruikt.